Assignment 2
HUL479
Algorithms in Ancient India:Answers

March 15, 2017

1. If the solution of the equation $ax + c = by$ is (α, β), prove the following. (20)
   (a) The solution of the equation $ax - c = by$ is $(b - \alpha, a - \beta)$
   (b) The solution of the equation $-ax + c = by$ is $(b - \alpha, \beta - a)$
   (c) The solution of $a.k.x + c.k = b.y$ is $(\alpha, k * \beta)$
   (d) The solution of $ax + ck = b.k.y$ is $(k.\alpha, \beta)$.
   (e) The solution of $ax \pm b.m + c = by$ is $(\alpha, \beta \pm m)$.

1A.1 Let the solution of the euqation $ax + c = by$ – (1)
be $x = \alpha, y = \beta$.
Hence $ab - (ax + c) = ab - by$
Or, $(a(b - x) - c = b(a - y)$.
I.e., $aX - c = bY$, where $X = b - \alpha$ and $Y = a - \beta$ is the solution.

1A.2 Let the solution of the euqation $ax + c = by$ – (1)
be $x = \alpha, y = \beta$.
Hence $(ax + c) - ab = by - ab$
Or, $a(x - b) + c = b(y - a)$.
Or, $-a(b - x) + c = b(y - a)$.
I.e., $-aX + c = bY$, where $X = b - \alpha$ and $Y = \beta - a$ is the solution.

1A.3 Let the solution of $a.x + c = b.y$ be (α, β).
Hence $a.k.\alpha + c.k = k.(a\alpha + c)$
$= k.(b\beta) = b.(k\beta)$
Thus $(\alpha, k.\beta)$ is the solution for $akx + kc = by$.
Similarly $b.k.\beta - c.k = k.(b\beta - c)$

$= k.(a\alpha) = a.(k\alpha)$
Thus $(k.\alpha, \beta)$ is the solution for $ax + kc = bky$.

1A.4 Let the solution for $ax + c = by$ be (α, β).
we have, $a\alpha + c = b\beta$
Hence, $a\alpha \pm b.m + c = b\beta \pm b.mb(\beta \pm m)$
Or, $(\alpha, \beta \pm m$ is the solution of $ax \pm bm + c = by$

2 2.1 $9x^2 + 52 = y^2$
9 is a perfect square. Dividing 52 by some optional number 4, we get (13 - 4)/2 = 9/2 and (13+4)/2 = 17/2, where 13 = 52/4.
So (x,y) = (9/(2*3), 17/2) is a solution. Taking optional number as 2, we get
(26-2)/(2*3) = 4, and (26+2)/2 = 14 as the values of (x,y).

2.2 $13 * x^2 - 13 = y^2$
One solution (1,0).
Solution for $13x^2 - 1 = y^2$ is:
$13 = 2^2 + 3^2$. Hence (1/2,3/2) and (1/3,2/3) are the possible solutions.
From this, solution for $13x^2 + 1 = y^2$ is:
(2 * 1/2 * 1/2, 13 * 1/4 + 9/4) = (3/2, 11/2)
Hence solution for $13x^2 - 13 = y^2$ is
x = 1 * 11/2+0 = 11/2, and
y = 13 * 1 * 3/2 + 0 = 39/2.
Next order solution is
x = 3/2 * 39/2 + 11/2 * 11/2 = 119/2, and
y = 13 * 3/2 * 11/2 + 11/2 * 39/2 = 429/2
Next order solution is
x = 3/2 * 429/2 + 11/2 * 119/2 = 649, and
y = 13 * 3/2 * 119/2 + 11/2 * 429/2 = 2340.

Hence (649,2340) is the solution.

2.3 Solution for $13x^2 + 13 = y^2$ is (3/2,13/2).
Solution for $13x^2 - 1 = y^2$ is (1/2,3/2).
Hence solution for $13x^2 + 1 =^2$ is
x = 2 * 1/2 * 3/2 = 3/2, and y = 13 * $(1/2)^2$ + $(3/2)^2$ = 11/2
Hence the solution for $13x^2 + 13 = y^2$ is
x = 3/2 * 11/2 + 13/2 * 3/2 = 18, and
y = 13 * 3/2 * 3/2 + 13/2 * 11/2 = 65
Hence the solution for $13x^2 + 13 = y^2$ is (18,65).

2.4 Solutions for
$2x^2 + 1 = y^2$ is (3,2)
$3x^2 + 1 = y^2$ is (2,1)
$5x^2 + 1 = y^2$ is (9,4)
$6x^2 + 1 = y^2$ is (5,2)
$7x^2 + 1 = y^2$ is (8,3)
$8x^2 + 1 = y^2$ is (3,1)

$10x^2 + 1 = y^2$ is (19,6)
$40x^2 + 1 = y^2$ is (19,3)
$90x^2 + 1 = y^2$ is (19,2)

3 For the equation $Nx^2 + 1 = y^2$, show that $x = \frac{2a}{N-a^2}$ is a possible value.

3A Consider the equation $Nx^2 + 1 = y^2$.
let a be some desired number.
Then, $(N - a^2)^2 + 4Na^2 = (N + a^2)^2$
Dividing both sides by $(N - a^2)^2$, we get

$1 + \frac{4Na^2}{(N-a^2)^2} = \frac{(N+a^2)^2}{(N-a^2)^2}$
i.e., $1 + N(\frac{2a}{N-a^2})^2 = \frac{(N+a^2)^2}{(N-a^2)^2}$
So $x = \frac{2a}{N-a^2}$ is a possible value.

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