as *variables* had been Aristotle and that was not in mathematics but in logic.[2]

Instead of modern algebra, the ancient Greeks and Vedic Indians possessed what has been called *geometrical algebra*, an algebra that makes use of geometrical methods and that we can only understand adequately if we desist from reading the expressions of modern algebra into it. The situation is similar to "Newton's equations" which I discussed in the 1995 issue of this *Journal* (23: 76) where I quoted the historian of science C. Truesdell: "It is true that we, today, can easily read them into Newton's words, but we do so by hindsight." We similarly tend to read modern equations into early medieval Indian algebra, which was just beginning to develop a symbolic notation, and are predisposed to do the same with respect to the much earlier phases of Vedo-Greek ritual geometry.

I shall begin with geometrical algebra, following Seidenberg. In the following figures, capital letters refer to points and line segments. I shall use small letters to refer to lengths and express their relationships through the language of modern algebra. In that language, the entire ancient edifice of geometrical constructions is reduced to one line: if the sides of the given rectangle are a and b, we are seeking a square with side c such that:

$$c^2 = a \cdot b \text{ and hence } c = \sqrt{a \cdot b}.$$

(So much for notation; to make it effective, we need to know how to extract a square root.)

In the Indo-Greek geometrical algebra, the construction of a square equal in area to a given rectangle is carried out in two steps. In the first, the rectangle is transformed into a difference of two squares. In the second, this difference is made equal to a square with the help of the Theorem of Pythagoras. Step 1 is illustrated by Figure 4 (from Seidenberg 1983: 99, which follows Thibaut's translation of Baudhāyana's *Śulbasūtra* 1.54).

The given rectangle is ABCD. Now carve out a square AEFD with sides equal to AD. The remainder is EBCF. Divide it into two equal rectangles, EGHF and GBCH. Place one of these on top: DFIJ. Fill the empty space in the corner by the small black square: FHKI. In so doing we have created a larger square: AGKJ. Because of the move of the hatched rectangle from the right to the top, it is easy to see that the irregular area without that small square, viz, AGHFIJA, has the same area as the original rectangle ABCD. The difference between the two

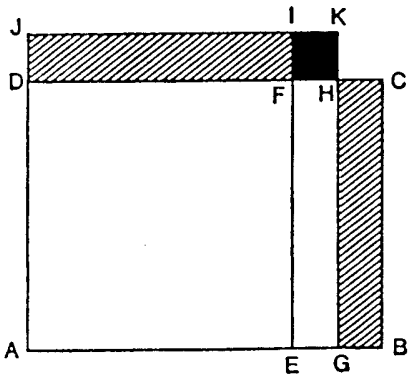


Figure 4. Transformation of a rectangle into a difference of two squares.

squares AGKJ and FHKI is the area of the original rectangle. End of Step 1.

To construct the difference of two squares geometrically (Step 2), use is made of the Theorem of Pythagoras as in Figure 5:

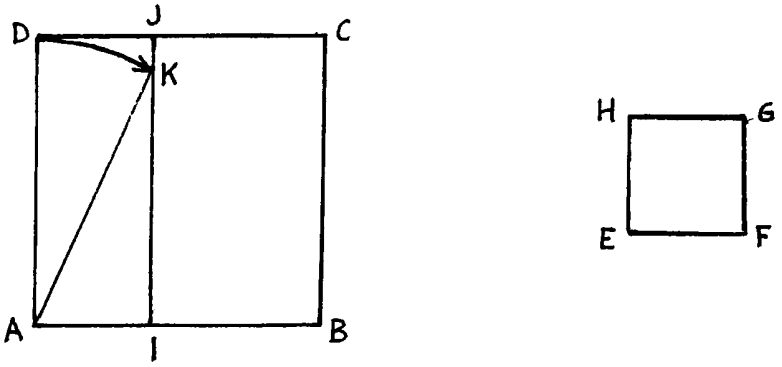


Figure 5. Constructing a square equal to the difference between two squares with the help of the Theorem of Pythagoras.

Since Seidenberg does not provide it, I shall improve Thibaut's translation of Baudhāyana's *Śulbasūtra* 1.51 (in Satyaprakash and Sharma 1968: 55) by following Hayashi 1995: 107 and interpret it (between square brackets) in terms of Figure 5: "One who wishes to deduct one square [EFGH] from another square [ABCD] should cut off a piece from the larger square with the side [AI] of that square one wishes to deduct [i.e., so that AI equals EF]. One should draw the longer side [AD] of the cut-off piece [AIJD] obliquely up to the

opposite side [making the arc DK]. One should cut that side [IJ] at the point [K] where it [AD] falls. By the line [IK] which has been cut off, the small square is deducted."

Baudhāyana divided the Theorem of Pythagoras into two complementary parts: proposition 1.51 relates to the difference between two squares; the addition had been formulated in 1.50 (corresponding to Euclid I.47). Note that Step 2 may be incorporated in Step 1 by "drawing" (in Figure 4) KG of the cut-off piece KGEI "obliquely" (*akṣṇayā*).

I have already expressed the problem in the simple language of modern algebra. If we combine Seidenberg with van der Waerden (1983: 11), we may depict its original Vedo-Greek complexity in algebraic terms, as follows. Refer to the base AB of the original rectangle of Figure 4 as a and to its height AD = BC as b. The area of that rectangle is $a \cdot b$. The area of square AEFD with sides equal to AD is b^2. The area of the remainder EBCF is $(a - b) \cdot b$. The area of each of the two equal rectangles EGHF and GBCH into which it is divided is $((a - b) \cdot b)/2$. After placing one of these on top, the area of the small square FHKI which fills the empty space in the corner is $((a - b)/2)^2$ or $(a^2 - 2ab + b^2)/4$. The area of the larger square AGKJ which has been created is $(b + (a - b)/2)^2$ or $((a + b)/2)^2$ or $(a^2 + 2ab + b^2)/4$. With the help of algebra, it is easy to see that the difference between the two squares is $a \cdot b$, i.e., the area of the original rectangle.

Now the deduction of the smaller square from the larger one. Call the side of the second b, that of the first a and that of the difference between the two that has to be found: c. If, in Figure 5, AI = b and AK = AD = a, IK is the side of the square, c, since: $c^2 = a^2 - b^2$ – an algebraic expression of Pythagoras' Theorem.

Seidenberg discovered that the same two steps were taken by Euclid in Book II of his *Elements*. Van der Waerden concludes:

> This is only one of several possible constructions. The fact that Euclid and the *Śulvasūtras* both use one and the same more complicated construction, based on the Theorem of Pythagoras, is a strong argument in favour of a common origin.

The Vedic geometrical rituals emphasized equality of *area* whereas the Greeks emphasized equality of *volume*. Since volume is more complex than area, this suggests that the Vedic geometry might have been the earlier of the two, or a more direct survival of the original. Is Vedic originality in the domain of ritual geometry likely in terms of what is known of Vedic history? Can it be evaluated from the perspective of the Vedas themselves?