Chomsky Hierarchy

- **Grammars are classified by the form of their productions.**
- **Each category represents a class of languages that can be recognized by a different automaton**
- **The classes are nested, with type 0 being the largest and most general, and type 3 being the smallest and most restricted.**

- **The classifications are:**

type 0		
	grammar	**unrestricted**
	language	**recursively enumerable**
	automaton	**Turing machine**
type 1		
	grammar	**context-sensitive**
	language	**context-sensitive**
	automaton	**linear-bounded automaton**
type 2		
	grammar	**context-free**
	language	**context-free**
	automaton	**pushdown automaton**
type 3		
	grammar	**regular**
	language	**regular**
	automaton	**finite-state automaton**

- **Type 0 Unrestricted**
  - $x \rightarrow y$
  - $x \in ((V \cup T)^* V (V \cup T)^*)$
  - $y \in (V \cup T)^*$
  - **LHS contains at least one non-terminal**
  - **generate recursively enumerable languages**
  - **recognized by Turing Machines**
  - **example (from Rich):**
    - **generates** $L = \{ww \mid w \in (a+b)^*\}$
    - **Generate** wCw^RE
      C is a temporary center marker
      E is a wall
    - **Reverse** w^R **by pushing symbols in** w^R**, leftmost first, over the wall with a pusher P**
    - **Clean up by removing C and** E

$V = \{S, T, C, P\}$
$T = \{a, b\}$
P :

1.	S	$\rightarrow$	TE
2.	T		aTa
3.		|	bTb
4.		|	C
5.	C	$\rightarrow$	CP
6.	Paa	$\rightarrow$	aPa
7.	Pab	$\rightarrow$	bPa
8.	Pba	$\rightarrow$	aPb
9.	Pbb	$\rightarrow$	bPb
10.	PaE	$\rightarrow$	Ea
11.	PbE	$\rightarrow$	Eb
12.	CE	$\rightarrow$	ϵ

– **Derive** aab

- **Type 1 Context-Sensitive**
  - $\boldsymbol{x \longrightarrow y}$
    or $\boldsymbol{S \longrightarrow \epsilon}$ and S is not in RHS.
  - $\boldsymbol{x \in ((V \cup T)^* V (V \cup T)^*)}$
  - $\boldsymbol{y \in (V \cup T)^+}$
  - $\boldsymbol{|x| \leq |y|}$ **– non-contracting**
  - **generate context-sensitive languages**
  - **recognized by linear-bounded automata**

– **example**

$V = \{S, A, B\}$
$T = \{a, b, c\}$
P :

1.	S	$\rightarrow$	abc
2.		$\mid$	$aAbc$
3.	Ab	$\rightarrow$	bA
4.	Ac	$\rightarrow$	$Bbcc$
5.	bB	$\rightarrow$	Bb
6.	aB	$\rightarrow$	aa
7.	aB	$\rightarrow$	aaA

generates $L = \{a^n b^n c^n \mid n \geq 1\}$
derive $a^3 b^3 c^3$

- **Type 2 Context-Free**
  - $x \longrightarrow y$
  - $x \in V$
  - $y \in (V \cup T)^*$
  - generate context-free languages
  - recognized by pushdown automata
  - example

$V = \{S\}$
$T = \{a, b\}$
$P:$

1.	S	$\longrightarrow$	aSb
2.		$\mid$	ϵ

generates $L = \{a^n b^n \mid n \geq 0\}$
derive $aabb$

- **Type 3 Regular**
  - **A regular grammar is right-linear or left-linear (but not both)**
  - $A, B \in V$
  - $y \in T*$
  - **Right-Linear productions have the form** $A \rightarrow yB$ **or** $A \rightarrow y$
  - **Left-Linear productions have the form** $A \rightarrow By$ **or** $A \rightarrow y$
  - **generate regular languages**
  - **recognized by finite-state automata**
  - **example**

$V = \{S\}$
$T = \{a\}$
$P:$

1.	S	$\rightarrow$	aS
2.		$\mid$	ϵ

generates $L = \{a^n; \mid n \geq 0\}$
derive aaa