Śivasūtra

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Given a class of sets, is it always possible to
arrive at a linear order with markers that will define all the sets?

Petersen provided an algorithm.

In what follows we discuss this algorithm.
But before that some Mathematical preliminaries.

Partial Order

A stict partial order is a binary relation R $(<)$ over a set P where R is Relflexive and transitive.

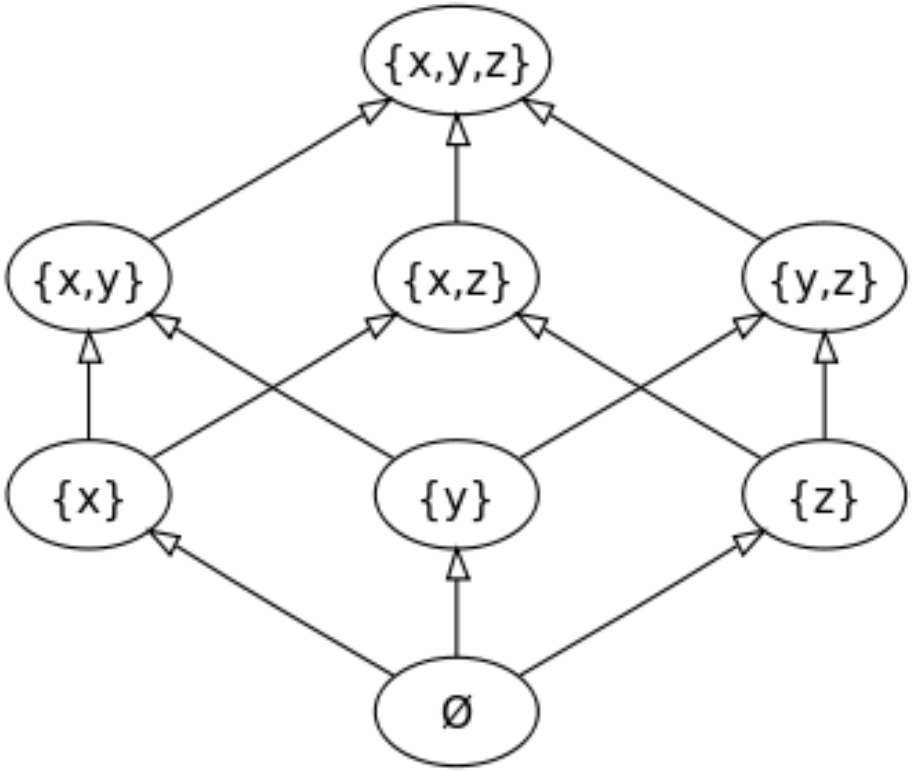
- A relation R is reflexive, if aRa.
- A relation R is transitive, if aRb, bRc then aRc.

Partially Ordered Set (POSET)

A set S is a partially ordered set (POSET), if R is a partial order over the elements of S.

The power set of S is a POSET over the relation of set inclusion/ subset relation.

In a POSET, all sets need not be comparable. Or in other words, there will be pair of sets which have partial overlap between them.



The Hasse diagram of the set of all subsets of a three-element set $\{x, y, z\}$, ordered by inclusion. Sets on the same horizontal level are incomparable with each other. Some other pairs, such as $\{x\}$ and $\{y,z\}$, are also incomparable.

S-alphabet

A well-formed Śivasūtra-alphabet or S-alphabet is a triple $(A, \Sigma, <)$ consisting of a finaite alphabet A, a finite set of markers Σ such that $A \cap \Sigma = \phi$, and a strict partial order $<$ on $A \cup \Sigma$.

S-encodable/pratyāhāra

A subset T of the alphabet A is S-encodable in an $S-$ *alphabet* $(A, \Sigma, \leq)$ iff there exists $a \in A$ and $M_1 \in \Sigma$ such that $T = \{b \in A / a \leq b < M_1\}$.
aM_1 is called a pratyāhāra or S-encoding of T in $(A, \Sigma, <)$.

S-Order and S-sortable

Given a set of classes, order the elements of the classes (without duplications) in a linear order (in a list) such that each single class forms a continuous interval with respect to that order.

- The target orders are called S-orders
- A set of classes is S-sortable if it has an S-order

S-Order and S-sortable

Given a set of classes, find an S-order of the elements of the classes. Interrupt this list by markers (anubandhas) such that each single class can be denoted by a sound-marker-pair (pratyāhāra).

Every S-order becomes a śivasūtra-alphabet (S-alphabet) by adding a marker (anubandha) behind each element.

Examples

- Given the set of classes {{a, b}, {a, b, c}, {a, b, c, d}}, the order **a b c d** is one of its S-orders and a $\{M_1 b M_2 c M_3 d M_4\}$ is one of its S-alphabets.
- The set of classes {{a,b},{b,c},{c,a}} is not S-sortable.
- The set of classes {{d,e},{a,b},{b,c,d},{b,c,d,f}} is not S-sortable.

Algorithm to get S-order

Algorithm to decide whether the given set of sets is S-encodable or not.

- Take the Union of all the sets. Let it be denoted by U.
  Let the given sets be denoted by A[i]
- Take all possible intersections of the given sets.
- Now we have the given sets, universal set, and the intersections of the sets.
  - Name these sets in the following way.
  - Name the smallest set containing a as 'a'.
  - If two sets have same name, say b and c, name it as 'bc'.
  - At the most only n sets will have names, if 'n' is the number of distinct elements in U. Remaining sets are names as 1,2,3 etc.

Algorithm to get S-order ...

- Now draw a graph as follows.
  - Draw a circle corresponding to each of the sets listed in step 3. Place the circles as described below.
  - Draw a circle corresponding to null set at the top, and a circle corresponding to the Universal set at the bottom.
  - Every set is connected to it's immediate subset by an upward arrow. Place the circles in such a way that there are no crossings of the arrows.
- If you can avoid the crossings altogether, then go to next step. In case you can not avoid crossings, duplicate the elements one at a time in order to avoid the crossings.

Algorithm to get S-order ...

- Remove all the arrows to the null set. Then select those arrows, that lead to a boundary of the graph towards its infinte side.
  If it contains all the nodes, then Shivasutra style representation exists for the give set of sets. The procedure to get the shivasutra style representation is described in step 7.
  However, if there are some nodes which are in the interior side of the boundary facing infinite side, then one can not have a shivasutra style representation corresponding to the given set of sets.

Algorithm to get S-order ...

- Traverse the graph as explained below.
  - Start from the universal set. You can traverse the graph either in clockwise or anticlockwise direction.
  - Go on writing the names of the nodes (which are member of universal sets; do not write the names 1,2,3 etc), as you are traversing them.
  - Insert a marker, when you traverse in an oppposite direction of an arrow. Then remove multiple occurances of the same node, keeping any one of its occurances.
  - Replace consecutive markers by a single marker.

  This should give the shivasutra style representation (linear representation) of the members of the sets, such that one can get all the given sets, just by refering to one of the members of the set followed by a marker.
- Multiple traverses of the graph are possible.

Examples

Decide if the following classes of sets are S-sortable, and if yes, give the S-order (linear order).

- {{b,c},{d,e,f},{a,d,f},{b,e,f}}
- {{a,b},{a,b,c},{a,b,c,d}}
- {{a,b},{b,c},{a,c}}
- {{a,b,},{b,c,d},{b,c,d,f},{d,e}}
- {{d,e},{a,b},{f,i},{g,h},{b,c,d,f,g,h,i},{c,d,e,f,g,h,i}}