A HISTORY OF *PIṄGALA*'S COMBINATORICS

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Abstract: *Combinatorics forms an important chapter in the history of Indian mathematics. The tradition began with the formal theory of Sanskrit meters formulated by Piṅgala in the 2nd century B.C.E. His recursive algorithms are the first example of recursion in Indian mathematics. Piṅgala's calculation of the binomial coefficients, use of repeated partial sums of sequences and the formula for summing a geometric series became an integral part of Indian mathematics. This paper systematically traces the extent to which Piṅgala's algorithms have been preserved, modified, adapted or superseded over the course of one and a half millennia. It also addresses Albrecht Weber's criticism about Halāyudha's attribution of the construction of what is now known as Pascal's triangle to Piṅgala. While agreeing with Weber's criticism of Halāyudha, this paper also faults Weber's interpretation of Piṅgala, but shows that the construction can still be traced to Piṅgala.*

1. INTRODUCTION

After giving an exhaustive account of Sanskrit meters in *Chandaḥśāstra* in the 2nd century B.C.E., *Piṅgala* concludes with a formal theory of meters. He gives procedures for listing all possible forms of an n-syllable meter and for indexing such a list. He also provides an algorithm for determining how many of these forms have a specified number of short syllables, that is, an algorithm for calculating the binomial coefficients ${}_nC_k$. What is striking is that this is a purely mathematical theory, apparently mathematics for its own sake. Unlike *Pāṇini*'s Sanskrit grammar, *Piṅgala*'s formal theory has no practical use in prosody. However, his mathematical innovations resonate with modern mathematics. *Piṅgala* consistently uses recursion in his algorithms, a tradition which stretches from *Pāṇini* (6th century B.C.E.) to *Āryabhata* (5th century C.E.) to *Mādhava* (14th century). *Āryabhata* provides a sine table which is the same as the table of chords of Hipparchus. Both tables rely on triogonometric identities for their construction. *Āryabhata* then proceeds to give a recursive algorithm for generating the same table. The algorithm is essentially a numerical procedure for solving the second order differential equation for sine and thus necessarily less accurate than the one based on triogonometric identities. *Mādhava* follows *Āryabhata* and develops the power series for sine, cosine and arctangent using recursion.

A remarkable example of the mathematical spirit of *Piṅgala*'s work is his computation of the powers of 2. He provides an efficient recursive algorithm based on what computer scientists now call the divide-and-conquer strategy. Another example is his formula for the sum of the geometric series with common ratio equal to 2. It was generalized to an arbitrary common ratio by *Śridhara* (c. 750 C.E.). Curiously, following *Piṅgala*, the formula for the geometric series is almost always combined with *Piṅgala*'s divide-and-conquer algorithm. Even in the 14th century, *Nārāyaṇa* in *Gaṇitakaumudi* repeats *Piṅgala*'s algorithm almost verbatim for summing geometric series. *Śridhara* also provided the modern day formula for calculating the binomial coefficients which replaced *Piṅgala*'s recursive algorithm in Indian mathematics. The prosodists still continued to use *Piṅgala*'s method. *Piṅgala*'s algorithms were generalized by *Sārṅgadeva* to rhythms which use four kinds of beats - *druta*, *laghu*, *guru* and *pluta* of durations 1, 2, 4 and 6 repectively (*Saṅgītaratnākara*, c. 1225 C.E.). In another direction, *Nārāyaṇa* developed many different series and their applications in *Gaṇitakaumudi* (see Kusuba[1]).

There is a mystery surrounding *Piṅgala*'s computation of $_nC_k$. It is almost universally accepted on the authority of *Halāyudha* (10th century, C.E.) that *Piṅgala*'s last *sūtra*, "*pare pūrṅaṃiti*", implies the construction of *meru prastāra* (what is now known as "Pascal's triangle") for computing $_nC_k$. However, except for *Virahāṅka*, none of the authors (from *Bharata* – 1st century C.E. onwards) before *Halāyudha* describes such a construction or even employs the designation *meru prastāra* for the construction they do describe. This is strange in view of the fact that the algorithms of *Piṅgala* have been copied and elaborated by later authors for more than a millennium after *Piṅgala*. Albrecht Weber[2] in 1835 commenting on *Halāyudha*'s interpretation flatly declares that "that our author (*Piṅgala*) may have had in mind something like *meru prastāra* does not follow from his words in any way". Alsdorf[3] in 1933 asserts that Weber's statement has no foundation and that Weber misunderstood *Halāyudha*. He then goes on to conjecture, without presenting any evidence, that the repetition of the *sūtra* "*pare pūrṅaṃ*" at the end of the composition is a later addition and that it was inserted as a reference to *meru prastāra* invented later. Authors from *Bharata* onwards do describe a construction which yields the same triangle. Instead of filling the triangle from the top, they do so diagonally from left to right. Although both constructions yield the same triangle, they are algorithmically quite different and in fact, *Virahāṅka* includes both with evocative names *sūci* ("needle") *prastāra* and *meru* ("mountain") *prastāra*. The method described in *Bharata*'s *Nātyaśāstra* is based on a technique very common among Indian mathematicians, namely, creating a new sequence from a given sequence by listing its partial sums. Indeed, summing sequences of partial sums became a standard topic in Indian mathematics. *Aryabhaṭa* provides a formula for the sum of sequences of partial sums of the sequence $1,2,3, \cdots,n$ which is the binomial coefficient $_{n+2}C_3$. The sum $1+2+\cdots+n$ is the binomial coefficient $_{n+1}C_2$. *Nārāyaṇa* in the 14th century gave a general formula in the form of binomial coefficients for summing sequences obtained by repeatedly forming partial sums an arbitrary number of times. Sequences of partial sums were crucial also in *Mādhava*'s development of infinite series. *Nātyaśāstra* evidently relies on the work of the past

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1 " Combinatorics and Magic Squares in India, A Study of Nārāyaṇa Paṇḍita's *Gaṇitakaumudi*, Chapters 13-14", Takanori Kusuba, Ph.D. Thesis, Brown University, May, 1993.
2 "Ueber die Metrik der Inder", Albrecht Weber, Berlin, (1863). Translation of quotes from Weber provided by Gudrun Eisenlohr and Dieter Eisenlohr.

3"Die Pratyayas. Ein Beitrag zur indischen Mathematik", Ludwig Alsdorf, Zeitschrift für Indologie und Iranistik, 9, (1933), pp. 97-157. Translated into English by S.R. Sarma, Indian Journal of History of Science, 26(1), (1991) masters and the construction must go back to *Piṅgala*. None of the prosodists following *Piṅgala* acknowledges *Bharata*, but they do acknowledge *Piṅgala*.

This paper systematically traces *Piṅgala*'s algorithms through the Indian mathematical literature over the course of one and a half millennia. It finds no evidence to support *Halāyudha*'s interpretation of *Piṅgala*'s last *sūtra*, but still traces the computation of the binomial coefficients to *Piṅgala*. Especially relevant are the compositions of *Bharata* and *Janāśraya* which are chronologically closest to *Piṅgala*. The section on Sanskrit meters in *Bharata*'s *Nāṭyaśāstra* (composed sometime between 2nd century BCE and 1st century CE) still has not been translated. Words are corrupted here and there and some of verses appear out of order. Regnaud[4] in his monograph on *Bharata*'s exposition on prosody concedes that a literal translation is not possible and skips many verses without attempting even a loose interpretation. In his 1933 paper on combinatorics in *Hemacandra*'s *Chandonuśasanam*, Alsdorf establishes a loose correspondence between *Bharata* and *Hemacandra* without translating *Nāṭyaśāstra*. The Sanskrit commentary of *Abhinavagupta* (c. 1000 CE) on *Nāṭyaśāstra* is spotty and frequently substitutes equivalent algorithms from later sources instead of explaining the actual verse. *Jānāśrayi* of *Janāśraya* (c. 6th century CE) is absent from the literature on *Piṅgala*'s combinatorics. Even in his otherwise excellent summary of Indian combinatorics before *Nārāyaṇa*, Kusuba barely mentions *Bharata* and does not mention *Janāśraya*. In this paper, we give translations of both works. Even in places where the literal text is unclear, its mathematical content is unambiguous. We also give translations of *Vṛttajātisamuccaya* of *Virahāṅka*, *Jayadevacchandaḥ* of *Jayadeva*, *Chandonuśasanaṃ* of *Jayakīrti* and *Vṛttaratnākara* of *Kedāra* which have not yet been translated into a western language. In the case of *Vṛttajātisamuccaya*, which was composed by *Virahāṅka* in *Prākṛta*, only its Sanskrit version rendered by his commentator is given. The survey in this paper is based on the following primary sources:

Date	**Title**	**Author**	**Translation**
c. 2nd century BCE	*Chandaḥśāstra*	*Piṅgala*	Included
2nd century BCE to 1st century CE	*Nāṭyaśāstra*	*Bharata*	Included
c. 550 CE	*Bṛhatsaṃhita*	*Varāhamihira*	Refer to Kusuba
c. 600 CE	*Jānāśrayī Chandovicitiḥ*	*Janāśraya*	Included
c. 7th century CE	*Vṛttajātisamuccaya*	*Virahāṅka*	Included
c. 750 CE	*Pāṭigaṇita*	*Śridhara*	Refer to Shukla
c. 850 CE	*Gaṇitasarasaṅgraha*	*Mahāvira*	Included
before 900 CE	*Jayadevacchandaḥ*	*Jayadeva*	Included
c. 950 CE	*Mṛtasañjīvanī*	*Halāyudha*	Refer to Weber
c. 1000 CE	*Chandonuśasanaṃ*	*Jayakīrti*	Included
c. 1100 CE	*Vṛttaratnākara*	*Kedāra*	Included
c. 1150 CE	*Chandonuśasanam*	*Hemacandra*	Included
1356 CE	*Gaṇitakaumudi*	*Nārāyaṇa*	Refer to Kusuba
Unknown	*Ratnamañjūṣa*	Unknown	Refer to Kusuba

[4] "La Métrique de Bharata", Paul Regnaud, Extrait des annals du musée guimet, v. 2, Paris, (1880).

The list does not include *Svayaṃbhū*'s *Svayaṃbhūchandaḥ* and *Chandaścityuttarādhyaya* (20th chapter) of *Brāhmasputasiddhānta* (628 CE) of *Brahmagupta*. The former does not cover combinatorics. Tantalizingly, *Brahmagupta*'s text does contain prosodist's technical terms like *naṣṭaṃ*, *uddiṣṭaṃ* and *meru*, but the text is too corrupted to make any sense out of it; even the commentator *Pṛthudaka* skips over it. The list also omits the *purāṇa*s because their description is essentially identical to descriptions in the sources listed above. Descriptions in *Agnipurāṇa* and *Garudapurāṇa* are identical and closely follow *Piṅgala*'s *sūtra*s except that they replace *Piṅgala*'s *prastāra* (method of listing meters) with a version given in *Nāṭyaśāstra*. They also assert that the "*lagakriyā*" (algorithm for computing ${}_nC_k$) is executed by means of the "*meru-prastāra*" without actually describing the algorithm. The description in *Nāradapurāṇa* is almost the same as the one in *Kedāra*'s *Vṛttaratnākara*.

Piṅgala's algorithms and their modifications by later authors are described in the next section. Translations of the relevant sections of the texts listed above are given in Section 3 to provide a detailed chronological history.

2. *PIṄGALA*'S ALGORITHMS

In the following, **G** denotes a long (Guru) syllable while **L** denotes a short (Laghu) syllable.

There are two classes of meters in Sanskrit:

1. *Akṣarachandaḥ*: These are specified by the number of syllables they contain. The vedic *akṣarachandaḥ*, referred to as *Chandaḥ*, are specified simply by the number of syllables. The later *akṣarachandaḥ*, called *Vṛttachandaḥ*, consist of 4 feet (*pāda*), each foot having a specified sequence of long and short syllables.

2. those which are measured by the number of *mātrā*s they contain. A *mātrā* is essentially a time measure. A short syllable is assigned one *mātrā* while a long syllable is assigned two. There are two kinds of these meters.
   (a) *Gaṇachandaḥ*: meters in which the number of *mātrā*s in each foot (*gaṇa*) is specified.
   (b) *Mātrāchandaḥ*: meters in which only the total number of *mātrā*s is specified.

Piṅgala's algorithms deal only with the *Vṛttachandaḥ*. These are of three types. *Sama* (equal) are the forms in which all four feet have the same sequence of short and long syllables. In *ardhasama* (half-equal), the arrangement of short and long syllables in the odd feet is different from that in the even feet, but each pair has the same arrangement. The forms which are neither *sama* or *ardhasama* are called *viṣama* (unequal). Although *Piṅgala* does list *gaṇachandaḥ* and *mātrāchandaḥ* that were employed in prosody at that time, much of their development came at a later time at the hands of *Prākṛta* prosodists. Each prosodist after *Piṅgala* has something to say about the combinatorics of *gaṇachandaḥ* and *mātrāchandaḥ*.

Sanskrit prosodists traditionally identify the following six formal problems and their solutions. (called *pratyaya*s). (*Piṅgala* does not assign any labels to them.)

THE SIX *PRATYAYA*S		
Name	**Literal Translation**	**Function**
Prastāra	Spread	Systematically lists all theoretically possible forms of a meter with a fixed number of syllables.
Naṣṭaṃ	Annihilated, Lost	Recovers the form of a meter when its serial number in the list is given.
Uddiṣṭaṃ	Indicated	Determines the serial number of a given form.
Lagakriyā	Short-Long-exercise	Calculates the number of forms with a specified number of short syllables (or long syllables).
Saṅkhyā	Count	Calculates the total number of theoretically possible forms of a meter.
Adhvayoga	Space measure	Determines the amount of space needed to write down the entire list of the forms of a meter

We now describe these in detail.

2.1 Prastāra

Piṅgala starts with the problem of listing all forms of an n-syllable meter. His recursive procedure is as follows. Start with the two forms of the 1-syllable meter: **G**, **L**. Then, to obtain a list of forms of an n-syllable meter, make two copies of the list of forms of the (n-1)-syllable meter. Append **G** to each form in the first copy and append **L** to each form in the second copy. For example, with n=1, n=2 and n=3, we get

n=1	n=2	n=3
G	**GG**	**GGG**
L	**LG**	**LGG**
	GL	**GLG**
	LL	**LLG**
		GGL
		LGL
		GLL
		LLL

Later prosodists always used modified versions of *Piṅgala*'s *prastāra*. Notice that the first column of the list consists of alternating **G**s and **L**s, the second column has alternating pairs of **G**s and **L**s, the third has alternating quadruples of **G**s and **L**s and so on. This provided an alternate way of constructing the list, column by column, from left to right. Another method is to notice the lexicographic order of the forms when read from right to left. Thus, the enumeration is begun by writing down *n* **G**s in the first line. Subsequently, scanning the latest form from left to right, write **L** below the first **G** in the form and copy the rest of the form after its first **G**. Fill the space before the **L** with **G**s. For example,

GGG
LGG
GLG

LLG
GGL
LGL
GLL
LLL

Prosodists later generalized these algorithms to list forms of *mātrā*-meters.

2.2 Naṣṭaṃ

The next algorithm determines the index (serial number) of a given form in the list. Notice that forms with an even index begin with **L** while those with an odd index begin with **G**. If we remove the first column, what remains is a list obtained by writing every form of the (n-1)-syllable meter twice. Therefore, index of the string of letters remaining after removing the first letter of a form is half of the original index if it is even and half of original index increased by 1 if it is odd. To write down the form corresponding to a given index *k*, write **L** as the first letter if *k* is even, **G** otherwise. Divide *k* by 2 if it is even, add one to it and divide by 2 if *k* is odd. This is now the index of the remaining string. Repeat the process until you have written down all the *n* letters. For example, if *k* = 6

6 is even, put **L** and halve it
3 is odd, add 1 before halving, put **G**
2 is even, put **L**, halve it.
1 is odd, add 1 before halving, put **G**

When you reach one, the algorithm produces a series of **G**s and is continued until the requisite number of syllables has been obtained. Thus, for meters of length 3, 4 and 5, you get **LGL**, **LGLG** and **LGLGG** respectively in the sixth position.

2.3 Uddiṣṭaṃ

Uddiṣṭaṃ is used to find the index of a given form. *Piṅgala*'s algorithm simply reverses the process used in *naṣṭaṃ*. Start with the initial value equal to 1. Scan the given form from right to left. At the first **L**, double the initial 1. From then on, successively double the number whenever an **L** is encountered. If a **G** is encountered, subtract one after doubling.

Example: **LGLG**

Initially, *k=1*.
Start at the last **L**: **LGLG,** get *k = 2*
The next letter is **G**: **LGLG,** Get *k = 2x2-1=3*.
The next letter is **L**: **LGLG,** Get *k = 2x3=6*.

Mathematician *Mahāvīra* replaced this algorithm by a method which amounts to an interpretation of the string as a binary number. Write down the geometric series starting with one with common ratio equal to 2 above the syllables of the form. (Effectively, he wrote down above each letter its positional value in terms of the powers of 2.) The index is then one plus the sum of the numbers above all **L**s in the string. This is the closest Indian mathematicians came to inventing binary numbers. Later prosodists with the exception of *Hemacandra* follow *Mahāvīra*. This method was probably known as far back as the 1st century. *Bharata*'s Verse (121) given in the Section 3.3.2 seems to outline this method.

2.4 Lagakriyā

Lagakriyā, answers the question: Out of all the forms of an n-syllable meter, how many have exactly *k* syllables? In modern notation: how to calculate the binomial coefficient ${}_{n}C_{k}$? All modern authors seem to have accepted *Halāyudha*'s attribution of the original algorithm to *Piṅgala*, but it is hard to see any connection between the words of the *sūtra* cited by *Halāyudha* and his interpretation that it is *Piṅgala*'s algorithm for calculating ${}_{n}C_{k}$. The *sūtra* by itself has no information for carrying out the construction described by *Halāyudha*. One of the main objectives of this paper is to settle this mystery by tracing the history of this algorithm through the available works of all the prosodists before *Halāyudha*. It will be shown that the original algorithm can be traced to *Piṅgala*, but not to the *sūtra* cited by *Halāyudha*, but another *sūtra* which he seems to omit or is unaware of. *Halāyudha* claims that the last *sūtra* of *Piṅgala* refers to *meru-prastāra* (Pascal's triangle). Here are *Piṅgala*'s last two *sūtra*s:

pare pūrṅaṃ "next, full"
pare pūrṅaṃiti "next, full, and so on"

The two *sūtra*s are identical except that the second has *iti* ("and so on") appended to it. *Halāyudha* correctly interprets the first *sūtra*. It is the formula: twice the number of forms of a meter equals the number of forms of the next meter. *Piṅgala* writes "full" instead of "twice" because the formula in the previous *sūtra* subtracts 2 from the doubled number. The word *pūrṅaṃ* tells us to restore the diminished double to its original full value. Even though the last *sūtra* is identical, *Halāyudha* inteprets it as an instruction to construct the *meru* (a mythical mountain) for calculating the binomial coefficients. He then gives detailed instruction for the construction:

"First write one square cell at the top. Below it, write two cells, extending half way on both sides. Below that three, below that again, four, until desired number of places (are obtained.) Begin by writing 1 in the first cell. In the other cells, put down the sum of the numbers of the two cells above it. ···"
Thus, for n=6, we get the following table:

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                  1
               1     1
            1     2     1
         1     3     3     1
      1     4     6     4     1
   1     5    10    10     5     1
1     6    15    20    15     6     1
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The numbers in the bottom row are the number of forms with 0, 1, 2, 3, 4, 5 and 6 **L**s respectively.

Weber who totally rejects *Halāyudha*'s attribution translates the two *sūtra*s together as:

"The following (meter comprises) the full (twice the sum of the combinations of the previous meter without subtracting 2)."

It is hard to argue against Weber's interpretation which is what the actual words are saying or to find evidence to support Alsdorf's argument in defense of *Halāyudha*. It is very likely that *Halāyudha* based his claim on an earlier source, but the only reference I could find is a mention in *Agnipurāṇa* and *Garudapurāṇa* which have the following half-verse:

pare pūrṅaṃ pare pūrṅaṃ meruprastāro bhavet|

"next full, next full, *meru prastāra* is created"

These *purāṇa*s are the only place where *Piṅgala*'s phrase, *pare pūrṅaṃ pare pūrṅaṃ*, appears again. Since *purāṇa*s are compiled perhaps by several authors over centuries, *Halāyudha*'s source probably goes quite far back, but seems lost. Still, I don't see any way to justify *Halāyudha*'s interpretation after reading all the available sources. The most reasonable interpretation of the repetition of the phrase *pare pūrṅaṃ* is that it is the standard Sanskrit usage for indicating a repeated action. So the two *sūtra*s together just mean the recursive formula $S_{n+1} = 2S_n$ where S_n is the total number of forms of the n-syllable meter.

I think the computation of binomial coefficients can still be traced back to *Piṅgala*. There is a *sūtra* quoted by Weber from the *Yajur* recension of *Piṅgala*'s *Chandaḥśāstra* which does exactly that. *Halāyudha* seems to be unaware of this, for it does not appear in his text. The *sūtra* in question is the following:

ekottarakramaśaḥ | *pūrvapṛktā lasaṃkhyā* || (23b)

Weber couldn't get any coherent sense out of this. He translates the *sūtra* as follows:

"Step-by-step always by adding one, the number of *la* is always combined with the previous (number)."

He then comments: "The 'number of *la* (referring to the last word *lasaṃkhyā* in the *sūtra*)' can only mean the number of combinations, which for each of the following meter is twice that of the preceeding. This meaning of '*la*' is no more provable than the required meaning (of '*la*') in Rule 22 (*sūtra* 8.22) as 'syllable'. Also the phrase *pūrvapṛktā* (in the *sūtra*) is not suitable to designate this doubling. Furthemore, the description of this doubling is expressly found in Rule 33 (*sūtra* 8.33) below. This mention here is strange. For the time being, I cannot see another explanation for the Rule 23b than the one given above. It is interesting to compare *Kedara*'s (6.2-3) description with this. ···." Weber then goes on to describe the *prastāra* algorithm given in *Kedāra*'s *Vṛttaratnākara* (verses 6.2, 6.3) which makes even less sense. Van Nootan[5] accepts this suggestion and offers *Kedara*'s method of enumeration as a clearer version of *Piṅgala*'s *sūtra*.

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[5] "Binary Numbers in Indian Antiquity", B. van Nooten, Journal of Indian Philosophy, 21, pp. 31-50, (1993).

I will try to show that this cryptic *sūtra* actually describes *lagakriyā*. It literally translates as follows:

ekottara = "increasing by one". *kramaśaḥ* = "successively, step by step, sequentially". *pūrvapṛktā* = "mixed with the next". *lasaṃkhyā* = "L-count".

In order to decipher this, consider a more detailed version in *Nāṭyaśāstra*:

ekādhikāṃ tathā saṃkhyāṃ chandaso viniveśya tu |
yāvat pūrñantu pūrveña pūrayeduttaraṃ gañaṃ || (124)
evaṃ kṛtvā tu sarveṣāṃ pareṣāṃ pūrvapūrañaṃ |
kramānnaidhanamekaikaṃ pratilomaṃ visarjayet || (126)
sarveṣāṃ chandasāmevaṃ laghvakṣaraviniścayaṃ |
jānīta samavṛattānāṃ saṃkhyāṃ saṃkṣepatastathā || (127)

"Put down (a sequence, repeatedly) increased by one upto to the number (of syllables) of the meter.
Also, add the next number to the previous sum until finished.
Also, after thus doing (the process of) addition of the next, (that is, formation of partial sums) of all the further (sequences),
Remove one by one, in reverse order, the terminal (number) successively.
Of all meters with (pre)determined (number of) short syllables
Thus know concisely the number of *sama* forms"

What is striking is the close correspondence between the key words in *Piṅgala* 's *sūtra* and the phrases in *Bharata*'s verses. *Piṅgala* 's *sūtra* begins with the word *ekottara* while *Bharata* begins with the word *ekādhikaṃ*[6]. Both terms are used by medieval mathematicians to indicate arithmetic series with common difference equal to one (*eka*). The rest of the first line in *Bharata*'s verses specifies the length of the sequence which *Piṅgala* does not mention explicitly.

Piṅgala 's compound *pūrvapṛktā* parallels *Bharata*'s phrase, *pūrvena pūrayet* in the second line and more closely, to its compound version, *pūrvapūrañaṃ* in the third line. *Piṅgala* and *Bharata* both employ the term *pūrva*. It has a multitude of meanings, but what makes sense in the present context is the interpretation "in front or next". *pṛktā* is the past passive participle of *pṛc* which means "to mix or to join". It modifies the noun *lasaṃkhyā*. *Bharata* employs the verb *pṝ* (to fill, to complete, to make full) and uses its derivatives *pūrayet* and *pūrañaṃ*. *pūrayet* is the causative (optative mode) of *pṝ* while *pūrañaṃ* is a derived noun meaning the act or the process of filling. In the context of arithmetic, both *pṛc* and *pṝ* may be interpreted to mean "to add".

Bharata's second line explains the process of *pūrvapūrañaṃ*: "*yāvat* (until) *pūrñaṃ* (completed) *tu* (also) *pūrveña* (with the next) *pūrayet* (fill) *uttaraṃ* (the previous) *gañaṃ* (sum)." The third line of his algorithm then employs *pūrvapūrañaṃ* repeatedly to construct sequences of partial

[6] The texts of Regnaud and Nagar have the word *ekādikāṃ* (numbers beginning with one) instead of *ekādhikāṃ* (numbers increasing by one) which is what Alsdorf has.

sums: "*evaṃ* (thus) *kṛtvā* (after doing) *tu* (also) *sarveṣāṃ* (of all, every) *pareṣāṃ* (further, beyond) *pūrvapūrañaṃ* (the process of adding the next).

Piṅgala uses *pūrvapṛktā* in place of *pūrvapūrañaṃ* and omits the detail of *Bharata*'s second line. He also employs *kramaśah* (step by step) instead of *Bharata*'s phrase *sarveṣāṃ pareṣāṃ* to indicate a repeated action. With these identifications, *Piṅgala* 's *kramaśah pūrvapṛktā* becomes equivalent to *Bharata*'s third line. It tells us to repeatedly calculate sequences of partial sums.

Clearly, *Piṅgala* 's designation *lasaṃkhyā* must refer to the number of forms with the specified number of **L**s, that is, *Bharata*'s last two lines. The whole *sūtra* may now be interpreted as saying: "The number of forms with the specified number of **L**s is obtained by the process of repeatedly calculating the partial sums of the intial sequence 1,2, ···,n." The detail about systematically stripping the terminal partial sums given in *Bharata*'s fourth line is missing.

Following *Bharata*'s recipe, with $n = 6$, we get the following table:

6					
5	15				
4	10	20			
3	6	10	15		
2	3	4	5	6	
1	1	1	1	1	1

Read from bottom to top, each column consists of partial sums of the sequence in the previous column. The last partial sum in each case is omitted. The numbers 6, 15, 20, 15, 6 and 1 along the diagonal are numbers of forms with 1, 2, 3, 4, 5 and 6 **L**s respectively.

The recursive procedure rests on the fact that the forms with *k* short syllables of an n-syllable meter may be obtained by appending **L** to the forms with ($k-1$) short syllables of the (n-1)-syllable meter and appending **G** to its forms with *k* short syllables. That is, ${}_nC_k = {}_{n-1}C_k + {}_{n-1}C_{k-1}$ if $k<n$. Of course, ${}_nC_n=1$. In *Piṅgala*'s scheme, this is represented as successive partial sums. The first squence is ${}_1C_1$, ${}_2C_1$, ${}_3C_1$, ··· ${}_nC_1$ which is the first column in the example above. The second sequence ${}_2C_2$, ${}_3C_2$, ${}_4C_2$, ··· ${}_nC_2$ consists of the partial sums of the first sequence except the last partial sum. Later authors began the process by initially putting down a sequence of 1's (symbolically, the sequence ${}_0C_0$, ${}_1C_0$, ${}_2C_0$, ··· ${}_nC_0$) instead of the sequence 1,2,3,···,n. What is depicted is just a rotated Pascal's triangle.

The first time we see the kind of construction described by *Halāyudha* is in *Virahāṅka*'s *Vṛttajātisamuccaya* (7th century). *Virahāṅka* gives two ways of constructing this triangle: the first is the method of partial sums described above and calls it *sūci prastāra* ("needle spread"). The second is the *meru* construction described by *Halāyudha* except that *Virahāṅka* starts with the top row consisting of two cells. He instructs us to construct a table with two cells in the first row, three in the second, four in the third and so on. (Each row is implicitly assumed to be placed below the one above with an offset so that each cell straddles two cells of the row below.) In the top row, write the numeral 1 in each cell. In each of the other rows, write 1 in the end cells and in the rest of the cells, write the sum of the two cells above it.

Mathematicians soon adopted the combinatorial problem of choosing k objects out of n as a standard topic in Indian mathematics and illustrated it with a wide variety of examples. *Varāhamihira* (*Bṛhatsaṃhitā*, *Adhyāya* 76, Verse 22) extended the procedure and outlined a method for listing the actual ${}_nC_k$ combinations. *Ratnamañjūṣa* quotes an algorithm by an unnamed author for listing the serial numbers of ${}_nC_k$ combinations. Beginning with *Śridhara* in the 7th century, mathematicians started using the modern fomula for calculating ${}_nC_k$:

$$ {}_nC_k = \frac{n(n-1)(n-2)\cdots(n-k+1)}{1\cdot 2\cdot 3\cdots k} $$

2.5 Saṅkhyā

Piṅgala uses recursion also in his fifth algorithm which calculates the number S_n of all possible forms of a meter. He notes that the number of forms of the n-syllable meter is twice the number of forms of the (n-1)-syllable meter. One can calculate S_n simply by repeated doubling, but *Piṅgala* gives a recursive method for calculating 2^n which rests on the fact that if n is even, $2^n=(2^{n/2})^2$. So the recursion is: if n is even, $2^n=(2^{n/2})^2$ and if n is odd, $2^n=2\cdot 2^{(n-1)}$. *Piṅgala* sets it up as follows:

dvirardhe| rūpe śūnyam| dviḥ śūnye| tāvadardhe tadguṇitam|
"two in case of half. (If n can be halved, write 'twice'.)
In case one (must be subtracted in order to halve), (write) 'zero'.
(going in reverse order), twice if 'zero'.
In case where the number can be halved, multiply by itself (that is, square the result.)"

For example, with $n = 6$:
Construct the first two columns in the table below and then going back up, construct the third:

n=6	twice	$(2\cdot 2^2)^2 = 64$
n=3	zero	$2\cdot 2^2 = 8$
n=2	twice	$2^2 = 4$
n=1	zero	2

Thus, total number S_6 of possible forms of length 6 is 64.

The prosodists later added another method: S_n is equal to the sum of all the numbers obtained by *lagakriyā*. More importantly, *Virahāṅka* extended the recursion to the computation of the number of forms of *mātrā*-meters: $S_n= S_{n-1}+ S_{n-2}$ which of course generates what is now known as the Fibonacci sequence.

Piṅgala abstracts the fact that the list of forms of an n-syllable meter contains lists of forms of all the meters with fewer syllables and presents it as the sum of a geometric series:

dvirdvūnaṃ tadantānām|

"twice two-less that (quantity) replaces (the sequence of counts) ending (with the current count)."

That is, $2S_n - 2 = S_1 + S_2 + \cdots + S_n$. More explicitly, $2\cdot 2^n - 2 = 2^1 + 2^2 + \cdots + 2^n$, a formula for summing a geometric series.

Mathematicians from *Śridhara* on incorporated geometric series as an integral part of mathematics. Their formula is always coupled with *Piṅgala*'s recursive algorithm for computing powers, attesting to the lasting influence of *Piṅgala* on Indian mathematics. For example, here is a quote from *Śridhara* in the 7th century:

viṣame pade nireke guṇaṃ same'rdhīkṛte kṛtiṃ nyasya |
kramaśo rupasyotkramaśo guṇakṛtiphalamādinā guṇayet || (94)
prāgvatphalamādyūnaṃ nirekaguṇabhājitaṃ bhaved gaṇitaṃ| (95-i)

"When the number of terms (of the series) is odd, subtract 1 from it and write 'multiply (by the common ratio)'. When even, write 'square' after halving it. (Continue this) step by step (until the number is reduced to zero). Starting with 1, step by step in reverse order, multiply (by the common ratio) and square the number (as the case may be), multiply the final result by the first term. (The result is the next term in the series.) The result obtained as above, diminished by the first term of the series and then divided by the common ratio diminished by one is the sum."

Thus, we get the fomula

$$a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r - 1}$$

Nārāyaṇa in the 14th century has the same formulation:

viṣame pade virūpe guṇaḥ same'rdhīkṛte kṛtiścāntyāt|
guṇavargaphalaṃ vyekaṃ vyekaguṇāptaṃ mukhāhṛtaṃ gaṇitaṃ||

"When the number of terms (of the series) is odd, subtract 1 from it and write 'multiply' (by the common ratio). When even, write 'square' after halving it. The result obtained by multiplying and squaring in reverse order starting from the last, diminished by one, divided by the common ratio diminished by one and (finally) multiplied by the first term is the sum."

2.6 Adhvayoga

The last algorithm is a strange computation of the space required to write down all the forms of a particular meter. Allowing for a space between successive forms, the space required is twice the number of forms minus one. Janāśraya sets the width of each line equal to the width of a finger and calculates that the space required to write down forms of the 24-syllable meter is 33,554,431 finger-widths or about 265 miles. Clearly not a practical proposition! One has to assume that the calculation is meant to demonstrate the immensity of the list.

3. TRANSLATIONS

In the following, the translations appear in quotes. Comments in parantheses are added to clarify the meaning. The numbering of the verses is the same as their numbering in the primary sources listed at the end of this section.

3.1 *PRASTĀRA*

3.1.1 Piṅgala

dvikau glau | (8.20) "Double **GL**-pair."

We get the following:

G
L
G
L

miśrau ca | (8.21) "(the pair of **GL**s) mixed and."

Mixing **GL**-pair with another **GL**-pair results in **GGLL**. This is then placed in the second column. The result is:

GG
LG
GL
LL

pṛthag (g?)lā miśrāḥ | (8.22) "Repeatedly **GL**s mixed."

GGG
LGG
GLG
LLG
GGL
LGL
GLL
LLL

vasavastrikāḥ | (8.23) "eight triples."

These are the 8 triples listed above. *Piṅgala* names (codes) these triples as **m**, **y**, **r**, **s**, **t**, **j**, **bh** and **n** respectively.

Piṅgala does not give a procedure for enumerating forms specified by fixing the number of *mātrā*s, that is, the *mātrāchandaḥ*, but merely lists the five possible forms of 4-*mātrā* feet, reflecting the fact that most of the development of *mātrāchandaḥs* occurred much later, mostly under the influence of *prākṛta* poetry. The five forms are given by the following *sūtra*.

gau gantamadhyādirnlaśca| (4.13) "**G**-pair **G**-end-middle-first **nL** and."

That is, **GG**, **LLG**, **LGL**, **GLL**, **LLLL**. Note **n** in **nL** means the triple **LLL** according to *Piṅgala*'s code.

Later algorithms adapted for *mātrā-chandaḥ*s produce the list in the same order.

3.1.2 Bharata

eteṣāṃ chandasāṃ bhūyaḥ prastāravidhisaṃśrayam |
lakṣaṇaṃ sampravakṣyāmi naṣṭamuddiṣṭameva ca || (112)

"I will now speak comprehensively of combinations (generated) by the *prastāra* procedure (or rule), the index (serial number), *naṣṭaṃ* and also *uddiṣṭaṃ* of these meters."

prastāro'kṣaranirdiṣṭo mātroktaśca tathaiva hi |
dvikau glāviti varṇoktau miśrau cetyapi mātrikau || (113)

"*prastāra* specified by syllables and similarly also indeed spoken of by *mātrā*.
'Double pair **GL**' spoken-of-syllable pair, 'mixed and', also *mātrika* pair."

The first line clearly refers to the two kinds of *prastāra*. The second line quotes *Piṅgala*'s first two *sūtra*s, "*dvikau glau*" and "*miśrau ca*", as a reference for syllable-based *prastāra* and then adds "*api mātrikau*" which seems to suggest that syllable pairs are to be replaced by *mātrika* pair. *Abhinavagupta* agrees, but adds that this should be done within the constraint of the specified numbers of *mātrā*s. He then describes a *mātrā-prastāra* which is essentially the one given by *Virahāṅka*. This is a procedure which is a modification of *Bharata*'s first *prastāra* given below (verses 114 and 115) which, in turn, is quite different from *Piṅgala*'s version. In particular, it does not follow the formula: "Double pair **GL**, mixed and". The term *mātrika* is usually taken to mean "containing one *mātrā*". For example, *Jayadeva* in the 10^{th} century writes: "*mātriko lṛjuḥ*" (1.3), "*mātrika* **L** (that is, **L** containing one *mātrā*) (is a) straight (line)". However, this interpretation does not seem to fit here. Another possibility is to interpret *mātrikau* to mean "turned into or adapted to *mātrā*" and try to adapt *Piṅgala*'s "*dvikau glau*, *miśrau ca*" construction to a *mātrā-prastāra*. To construct the *prastāra* of a n-*mātrā* meter, instead of appending **G**s and **L**s to two copies of the *prastāra* of the (n-1)-syllable meter, we are forced to append **L** to the *prastāra* of (n-1)-*mātrā* meter and append **G** which is worth 2 *mātrā*s to the *prastāra* of the (n-2)-*mātrā* meter. With this, we get the following:

n=1

L

n=2

G
LL

n=3

LG
GL
LLL

n=4

GG
LLG
LGL
GLL
LLLL

and so on. Notice that the list for *n=4* is exactly the one given by *Piṅgala*.

The verses (114) and (115) below describe the first version of *Bharata*'s *prastāra* while the verse (116) describes the second.

guroradhastādādyasya prastāre laghu vinyaset |
agratastu samādeyā guravaḥ pṛṣṭhtastathā || (114)
prathamaṃ gurubhirvarṇairlaghubhistvavasānajam |
vṛttantu sarvachandassu prastāravidhireva tu || (115)

"Below the first **G** in the *prastāra*, put down **L**
Further (i.e. after **L**) also, the same (as the combination above) to be bestowed, but **G**s behind (i.e. before **L**) thus."

"The first form (is filled) with syllable **G**s, but the last with **L**s.
Thus (is) the *prastāra* procedure in the case of all meters."

The enumeration is begun by writing down *n* **G**s in the first line. Subsequently, to write down the next combination, write **L** below the first **G** in the line above and copy the rest of the line after its first **G**. Fill the space before the **L** with **G**s. For example,

GGG
LGG
GLG
LLG
GGL
LGL
GLL
LLL

An alternate version is:

gurvadhastāllaghuṃ nyasya tathā dvidvi yathoditam |
nyasyet prastāramārgo'yamakṣaroktastu nityaśaḥ || (116)

"**G** below **L** to be put down thus repeatedly doubled as has been said put down this course of *prastāra* (when) syllable-specified, always."

In the first column, write down **G**s alternating with **L**s. In the second column, write two **G**s alternating with two **L**s. In the third column, write four **G**s alternating with four **L**s. Continue to fill the successive columns by double the number of **G**s alternating with double the number of **L**s . This produces the following:

GGGG . . .
LGGG . . .
GLGG . . .
LLGG . . .
GGLG . . .
LGLG . . .
GLLG . . .
LLLG . . .
.
.
.

Once the required number of syllables is reached, it is clear how many rows are to be retained. For example, for meters of length of two, only two columns are needed and only four rows need to be retained since subsequent rows are copies of the first four.

mātrāsaṃkhyāvinirdiṣṭo gaṇo mātrāvikalpitaḥ |
miśrau glāviti vijñeyau pṛthak lakṣyavibhāgataḥ || (117)

"A foot (*gaṇa*) specified by the number of *mātrā*s is a mātrā-combination
Knowing 'mixed GL', (apply) repeatedly according to intended apportionment."

This verse may be just to reinforce what is indicated in verse (113). Alternately, it suggests that the first method may be used for *mātrā*-meters as well provided that syllables are adjusted to get the correct number of *mātrā*s. This is how the later prosodists adjust this algorithm to list forms of a *mātrā*-meter.

mātrāgaṇo guruścaiva laghunī caiva lakṣite |
āryāṇāṃ tu caturmātrāprastāraḥ parikalpitaḥ || (118)

"(Forms of) a *matrā*-foot (consiting of) **G** and two **L**s, having been attended to,
also (obtain) (the meter) *Āryā*'s four-*matra-prastāra* by combinations."

The meaning is clear. *Āryā* uses feet consisting of four *mātrā*s.

gītakaprabhṛtīnāntu pañcamātro gaṇaḥ smṛtaḥ |
vaitālīyaṃ puraskṛtya ṣaṇmātrādyāstathaiva ca || (119)

(meters) *Gītaka*'s and *Prabhṛtī*'s five-*mātrā* feet also, (as) recounted, (meter) *Vaitālīya* placed in front (i.e. beginning with *Vaitālīya*), (feet classes) beginning with 6-*mātrā* (feet) similarly, and."

Here, clearly an open-ended list of feet of increasing lengths is indicated.

tryakṣarāstu trikā jñeyā laghugurvakṣarānvitāḥ |
mātrāgaṇavibhāgastu gurulaghvakṣarāśrayaḥ || (120)

"Three-syllable (feet), triples, known by **L**-**G** syllable combination.
Mātrā feet classes also syllables **G**-**L** dependent."

3.1.3 Janāśraya

(*Janāsraya* denotes long syllables as 'bha' and short syllables as 'ha'. After stating each sūtra, *Janāśraya* also provides a commentary.)

prastāraḥ (6.1) "*Prastāra*"

ādye bhā (6.2) "At the beginning, **G**s.>

sarveṣāṃ chandasāmādye vṛtte gurava eva bhavanti | *etaduktaṃ bhavati* | *sarveṣāṃ chandasāmāñca vṛttaṃ sarvagurviti* |

"Only **G**s occur in the first form (in the listing) of all meters. This is what is said. All **G**s in the first form of all meters. (**GGGG…**)"

ādau hā (6.3) "In the first place, **L.**"

sarvaguruvṛttamādyaṃ kṛtvā tato dvitīyasya vṛttasya chedane dvitīyavṛttādau laghurekaḥ kārya iti |

"After creating all-**G** first form, then, dividing the second form (into two parts), place one **L** at the beginning of the second form."

pūrvavaccheṣaṃ (6.4) "The remainder (the rest) as before."

dvitīyavṛttādau laghumekaṃ kṛtvā tato'nantaramupari nyastaṃ pūrvavṛttamiva śeṣaṃ kuryāt | *śeṣamiti kriyamāṇavṛttaśeśaṃ* | *idaṃ prastāranetradvisūtrayuk sarveṣāṃ chandasāṃ yuk vṛttaprastāre yojyaṃ* |

"After placing one **L** at the beginning of the second form, then, next (to **L**), the remainder is made up just like the previous form placed above. 'remainder' means making up the remainder (after the first **L**) of the form. Such a two-piece union as *prastāra* lead (leading piece, **LGGG…**) must be employed in even (numbered) forms of the list in the case of all meters."

bhāhatulyāḥ (6.5) "**GL** to be balanced."

atha tṛtīyavṛttasya chedane pūrvavaccheṣaṃ kṛtvā parisamāpte dvitīye vṛtte'tha tṛtīyasya chedane tṛtīyavṛttādau guru dvitīyavṛttādivat tulyasṃkhyaḥ kāryaḥ |

"Now, dividing the third form (into two parts), as before, make the remainder as the completion of the second form, thus, in the division of the third form, in the beginning of the third form, **G**, (then), put (copy) the requisite number of syllables like (from) the second form. (**GLGG…**)"

iti ha (6.6) "Thus **L**."

atha tṛtīyavṛttādau pūrvavṛttalaghusṃkhyaṃ guruṃ kṛtvā teṣāṃ guruṇāmanantaraṃ ha iti laghurekaḥ kārya iti |

"Thus, at the beginning of the third form, convert the sequence of **L**s in the previous form into **G**, after those **G**s, place a single **L**. (This instruction applies to all the forms from now on.)"

śeṣaṃ pūrvavat (6.7) "remainder as before."

śeṣamiti kriyamāṇavṛttaśeṣamityarthaḥ | pūrvavaditi anantaramuparinystavṛttasūtravadityarthaḥ | etaduktaṃ bhavati | anantaramatītavṛttalaghugurusaṃkhyaṃ gurulaghuvinyāsaṃ kuryāditya- rthaḥ | idam tṛtīyaprastāranetraṃ trisūktayuktaṃ sarveṣāmyugvṛttaprastāre yojyaḥ | ābhyāṃ prastāranetrābhyāṃ kramaśo vivartamānābhyāṃ sarveṣāṃ chandasāṃ caturthādiṣu yugayugvṛtteṣu prastāravidhiḥ kāryaḥ | āsarvalaghuvṛttadarśnādeṣa sarveṣāṃ chandasāṃ prastāravidhiḥ |

" 'remainder' means making up the remainder of the form. 'as before' means the piece next (to the just written **L**), from the form placed above. This is what is said. Put down **G L** (syllables) counting up **L G** (syllables) of the form above next (to the just written **L**). (In the next sentence, I think *trisūktayuktaṃ* should be *trisūtrayuktaṃ* meaning union of 3 pieces (cf. 6.4). Then, the sentence translates as:) this three-piece union leading the third form should be employed for all odd numbered forms. The *prastāra* procedure of all meters should be performed by having these two *prastāra* leading terms alternately in even and odd forms, beginning with the fourth form, until the all-**L** form is seen, this (is) the *prastāra* procedure of all meters."

ayamanyaḥ krama|
"Here is another procedure."

bhāhau miśrāvadho'dhaḥ (6.8) "**G L,** mixed, (repeatedly) one below the other."

didṛkṣitasya chandasaḥ vṛttānāṃ pramāṇasaṃkhyā labdhā yāvattāvattau gurulaghumiśre ekāntaritau kāryāvadho'dhaḥ |

"After obtaining the total number of forms of a desired meter, mixing long and short (syllables), do (put down) as many alternating (**G** and **L**) pairs (as the total number of forms.)"

bhau bhau pare (6.9) “two **G**s, two **G**s, next.”

teṣāṃ tatha nyastānāṃ parato dvau gurulaghū cādho’dhaḥ kāryau | pūrvamekāntaritanyasta-gurulaghusaṃkhyāpramāṇaprāpteriti tadviparīta ityanuvartate | tatha nyastānāṃ teṣāṃ parato dviguṇaṃ dviguṇaṃ guravo laghavaścādho’dhaḥ kāryāḥ | āchandokṣarapramāṇaparisamāpte-rityuktamevarthaṃ nirūpayiṣyāmaḥ | ādyā dviguṇakriya catvāro guravaśca laghavaśca tato’ṣṭhāveva dviguṇadviguṇavṛddhiṃ sarveṣāṃ chandasāmācchandokṣaraparisamāpteriti |

“After putting down those (referring to 6.8), put down next pairs of **G**s and pairs of **L**s, repeatedly one below the other. (In the next sentence, the exact meaning of tadviparīta is unclear, but it says something like:) Having obtained the total number from the alternating **GL**s put down earlier, (the new column) follows the previous form modified. Having put these down thus, put down repeatedly doubled **G**s and **L**s, one below the other (in successive columns.) This procedure is completed when the number of syllables is reached, (the number of columns equals the number of syllables); we wish to indicate that this is the meaning of what is said. Doubling in the beginning, four **G**s and four **L**s, and then, eight of the two, repeatedly increased by doubling, completed when the number of syllables is reached, (thus) for all meters.”

3.1.4 Virahāṅka

ye piṅgalena bhaṇitā vāsukimāṇḍavyacchandaskārābhyāṃ |
tataḥ stokaṃ vakṣyāmi chātodari ṣaṭprakāraiḥ || (6.1)

“O Slim-waisted, I will describe the gist of what was taught by *Piṅgala* and the prosodists *Vāsuki* and *Māṇdavya* by means of six procedures.”

prastārā ye sarve naṣṭoddiṣṭaṃ tathaiva laghukriyām |
saṃkhyāmadhvānaṃ caiva chātodari tatsphutaṃ bhaṇāmaḥ || (6.2)

“O Slim-waisted, we reveal all those, *prastāra*, *naṣṭaṃ*, *uddiṣṭaṃ*, *laghukriyā*, *saṃkhyā* and *adhvā*.”

sūcirmerupatākāsamudraviparītajaladhipātālāḥ |
tathā śālmaliprasṭāraḥ sahito viparītaśālmalinā || (6.6)

“Thus, *Sūci, Meru, Patākā, Samudra, Viparīta-jaladhi, Pātāla, Śālmali and Viparīta-śālmali prasṭāra*.”

Virahāṅka describes 8 different ways of enumerating meter variations. The first two are the two versions of *Lagakriyā*. *Sūci* is *Piṅgala*’s *Lagakriyā* while *meru* construction is exactly what is now called the Pascal’s triangle. *Patākā* and *Samudra* are the two methods of listing all the forms of a meter given by *Bharata*. *Virahāṅka* also tells us how to modify these in order to enumerate the forms of *Mātra*meters. *Viparīta-jaladhi* is the procedure for listing the forms in reverse order. *Pātāla* lists the total number of permutations of a meter, the combined number of syllables in all the permuations, the combined number of *mātrā*s, the combined number of **L**s and the combined number of **G**’s. *Śālmali* tabulates in each line the number of **L**s, the number of syllables and the number of **G**'s in each of the variaions of a Mātra-vṛtta. *Viparīta-śālmali* produces the same table in reverse order.

maṇiravamālākāro dviguṇadviguṇairvardhitaḥ kramaśaḥ |
sthāpayitavyaḥ prastāro nidhanārdhamaṇīravārdhaśca || (6.13)

(*Patākā*) "*Prastāra* is established by (first) making a garland of **G**, **L** (that is, create a column of alternating **G**, **L**.), (then) repeatedly doubling step-by-step, (that is, the second column of alternating pairs **GG, LL** the third column of alternating quadurplets of **GGGG**, **LLLL** and so on), half of the syllables put down are **G**, and half are **L**."

dvitīyārdheśu kvāpi dīyate sparśo'pyantimaśchate |
teneyaṃ prastāre vṛttānāṃ kriyate gaṇana || (6.14)

"O slim one, in the second half, place **L** everywhere as the last syllable. In this way, construct the *prastāra* of *vṛtta*-meters."

ratnāni yathecchayā sthāpayitvā mugdhe sthāpaya prastāraṃ |
tāvacca piṇḍaya sphutaṃ sparśāḥ sarve sthitā yāvat || (6.15)

(*Samudra*) "O bewildered! After placing as many **G** as required, establish the *prastāra* by assembling (forms) until (the form with) **L** placed is seen everywhere."

prathamacamarasyādhaḥ sparśaḥ purato yathākrameṇaiva |
marge ye pariśiṣṭāḥ kaṭakaistān pūraya || (6.16)

"Successively (line-by-line), (put down) **L** below the first **G**; after it, the same as the line above; fill the remaining portion of the line with **G**s."

eṣa eva prastāro mātrāvṛttānāṃ sādhitaḥ kiṃtu |
mātrā yatra na pūryate prathamaṃ sparśaṃ tatra dehi || (6.20)

"*prastāra* of *mātrā*meters also is achieved similarly, except that whenever a form does not have the full complement of *mātrā*s, supply (the necessary number of extra) **L**s at the beginning of the form."

By this method, we get the following forms for the 4-mātrā foot:

GG
***L*LG**
***L*GL**
GLL
***L*LLL**

Here, the italicized ***L*** is the **L** supplied to make up for the mātrā deficit.

3.1.5 Mahāvīra

chandaśśāstroktaṣaṭpratyayānāṃ sūtrāṇi

"*sūtrā*s pertaining to six algorithms as announced in *chandas*-śāstra."

saṃkhyā viṣamā saikā dalato gurureva samadalataḥ |
syāllaghurevaṃ kramaśaḥ prastāro'yaṃ vinirdiṣṭaḥ || (5.334)

"Sequentially (starting with one and going upto the *saṃkhyā*), if the number is odd, add one, divide by two and let it be **G**. If it is even, divide by two and let it be **L**. (Continue until the requisite number of syllables has been written down.) This describes the method of enumeration."

Mahāvīra does not see any need for *Piṅgala*'s separate *prastāra* algorithm. He applies the *naṣṭaṃ* algorithm to numbers from one to the total number of forms to generate the full list of forms of a meter.

3.1.6 Jayadeva

prastāro naṣṭamuddiṣṭamekadvitrilaghukriyā |
saṃkhyā caivādhvayogaśca ṣadvidhaṃ chanda ucyate || (8.1)

"A meter is associated with six procedures, (namely) *prastāra, naṣṭaṃ, uddiṣṭaṃ, ekadvitrilaghukriyā (lagakriyā), saṃkhyā* and *adhvayoga.*"

sarvatraivacchandasi saṃsthāpyādau samastaguru vṛttaṃ |
ādyasya tatra guruṇo laghu kṛtvādhaḥ samaṃ śeṣaṃ || (8.2)

"In the case of all meters, put down at the start the form with **G**s everywhere. Then, below the first **G**, write **L** and the rest (after it) as (what is) above."

vidhimetameva kuryādbhūyo'pyādiṃ ca pūrayedgurubhiḥ |
iti yāvatsarvalaghu prastāre vṛttavidhireṣaḥ || (8.3)

"And follow this procedure repeatedly. Fill in the initial portion (before the **L**) with **G**'s until (you have reached) the form consisting of all **L**s in the enumeration. This is the procedure (for enumerating all forms.)"

3.1.7 Jayakīrti

gaṇānāṃ pratyayānāṃ ca mukhyaḥ prastāra eva saḥ |
tasmātprastārasūtraṃ taddhyekaṃ sarvatra dṛśyate || (8.1)

"Among algorithms for (meter) feet, the first is *prastāra*. Its *sūtra* is seen to be the same everywhere."

guroradhastāllghumāditaḥ kśipetparaṃ likhedūrdhvasamaṃ punastathā |
pāścātyakhaṇḍaṃ guruṇā prapūrayedyāvatpadaṃ sarvalaghutvamāpyate || (8.2)

"First, write **L** below the first **G**, followed by a copy of what is above. Fill the initial portion (before the newly written **L**) with **G**s. Repeat this until the form with all **L** is obtained."

vinyasya sarvagurvekadvicatuṣkānsamārdhasamaviṣamāṃhrīn |
prastārayetpṛthakpṛthagiti kramāt vṛttavidhirayaṃ prastāraḥ || (8.3)

"After writing down one, two or 4 feet consisting of **G**s in the case of *sama, ardhasama* or *viṣama* meter, enumerate by repeating thus (the procedure described above) step by step. This is the *prastāra* for the entire meter."

ekaikenāntaritā prastāraprathamapaṅktiriha gurulaghunā |
tattaddviguṇitagalataḥ kramāddvitīyādipaṅktayo'ntaritāḥ syuḥ || (8.4)

(Alternate *prastāra*) "The first column of the enumeration consists of alternating **G** and **L**. The other columns beginning with the second are filled by alternating groups of **G**s and **L**s, the number of syllables in each group twice the number in the previous column."

ardhasamaprastāre saṃdṛśyante samārdhasamavṛttāni |
viṣamaprastāre'tra tu samārdhasamaviṣamavṛttarūpāṇyakhilaṃ || (8.5)

"The *ardhasama* forms enumerated in this way include the *sama* forms. The *viṣama* forms include all the *sama* and *ardhasama* forms."

jātīnāmapi caturaḥ pādānvinyasya saṃbhavatsarvagurūn |
prastārayediti prāggaṇasaṃbhavamapi laghuprayogaṃ jñātvā || (8.6)

"Even in the case of mātrāmeters, the clever after writing down feet with all **G**'s, carries out *prastāra* knowing that (they have to) occasionally employ an initial **L**."

3.1.8 Kedāra

pāde sarvagurāvādyāllaghuṃ nyasya guroradhaḥ |
yathopari tathā śeṣaṃ bhūyaḥ kuryādamuṃ vidhiṃ || (6.1)

ūne dadyād gurūṇyeva yāvatsarvalaghurbhavet |
prastāro'yaṃ samākhyātaśchandovicitivedibhiḥ || (6.2)

(forms of a single foot). "Below the first **G** of the foot consisting of all **G**s, put down **L**. Repeatedly, make the rest same as what is above. This is the procedure. Supply **G**s when missing (syllables) (i.e. before the **L**.) (Continue) until (the form consisting of) all **L**s is created. The experts in *Chandaḥśāstra* call this *prastāra*."

3.1.9 Hemacandra

atha prastārādayaḥ ṣaṭ pratyayāḥ || (8.1)

"Now the six algorithms beginning with *prastāra*."

prākkalpādyago'dho laḥ paramuparisamaṃ
prāk pūrvavidhiriti samayabhedakṛdvarjaṃ prastāraḥ || (8.2)

"Below the first **G** of the previous form, (place) **L**. Beyond, the same as above. Before (the just written **L**), same procedure as before (i.e. write **G**s). Avoid (forms) which differ from the rules (For example, when treating a*rdhasama* forms by this method, avoid *sama* forms which are also created during the process.) (Thus is) *prastāra*."

glāvadho'dho dvirdvirataḥ || (8.3)

"The pair **G L** repeatedly (copied) one below the other. Thereafter, double repeatedly."
(This is the alternate version: The first column consists of alternating **G L**, the next alternating **GG LL** and so on.)

3.2 *NAṢṬAṂ*

3.2.1 Piṅgala

lardhe|(8.24) "In case of half, **L**."

In case the given number can be halved, put **L**.

saike g| (8.25) "In case (combined) with one, **G**."

In case it is necessary to add 1 in order to halve, put **G**.

Example: $n = 6$

6 is even, put **L** and halve it
3 is odd, add 1 before halving, put **G**
2 is even, put **L**, halve it.
1 is odd, add 1 before halving, put **G**

When you reach one, the algorithm produces a series of **G**s and is continued until the requisite number of syllables has been obtained. Thus, for meters of length 3, 4 and 5, you get **LGL**, **LGLG** and **LGLGG** respectively in the sixth position.

3.2.2 Bharata

vṛttasya parimāñantu chitvārdhena yathākramam |
nyasellaghu tathā saikaṃ akṣaraṃ guru cāpyatha || (128)

“according to the rule, when dividing the meter’s measure (serial number) into half, put down **L**, but with one (that is, if one is added to be able to halve the number), syllable **G**.”

3.2.3 Janāśraya

naṣṭaṃ (6.10) *“naṣṭaṃ”*

naṣṭamidanīṃ vakṣyate | hṛtvā hordhaṃ yasya kasyacit chandasaḥ saṃbhavati daśa śataṃ sahasratamaṃ vā darśayetyukte tenoktā yā saṃkhyā tāvanti rūpāṇi vinyasya tadardhamapanīya laghumekaṃ nyasya punaḥ punarevaṃ kāryaḥ |

“Now, let *naṣṭaṃ* be told. Dividing into half, (put down) **L,** when asked to show which of the forms occurs as the tenth or the hundredth or the thousandth (in the listing), Putting down the digits of the given number, obtain its half, put down one **L** and repeat again and again.”

datvaikaṃ same bhāḥ (6.11) “When (made) even after providing one, **G**s.”

ardhe punarhniyamaṇe yadi viṣamatā syāt tata evaikaṃ dattvā samatāṃ kṛtvā tato’rdhamapanīya gurumekaṃ vinyasya punaḥ punarevaṃ kāryaḥ | yāvaddidṛakṣitasya chandaso’kṣarāṇi paripūrṇāni bhavanti tāvadevaṃ kṛtvā idaṃ tadvṛttamiti darśayet |

“If during repeated halving, there occurs oddness (odd number), then creating evenness by giving (adding) one, after that, obtain half, put down one **G** and repeat again and again. Continue until the desired syllables of the meter have been competed, thus that form is to be exhibited.”

3.2.4 Virahāṅka

etāvatyaṅke kataradvṛttamiti naṣṭaṃ bhavati |
tajjānīhi vṛttaṃ katame sthāna ityuddiṣṭaṃ || (6.30)
viṣamāṅkeṣu ca cāmaraṃ sameṣu sparṣaṃ sthāpaya vṛttānāṃ |
ardhamardhamavaṣvaṣkate naṣṭāṅke sarvakaṭakāni || (6.31)
yatra ca na dadāti bhāgamekaṃ datvā tatra piṅdaya |
bhāge datte ca sphuṭaṃ mṛgākṣi naṣṭaṃ vijānīhi || (6.32)

“*Naṣṭaṃ* is (the question): What is the form, given such a number (that is, its serial number)? (Conversely), *uddiṣṭaṃ* is: if you know the form, what is its place (in the listing)?
When the form’s number is odd, put down **G**, **L** when even. (The first compund word of the second line is somewhat garbled, but the commenrary makes the meaning clear.) Halving repeatedly until number 1 is reached, (fill the the rest of the form with) all **G**s. (In the above procedure) whenever it is not possible to halve, (that is, the number is odd), add one and then halve. Know the true *naṣṭaṃ*, O deer-eyed!”

3.2.5 Mahāvīra

naṣṭāṅkārdhe laghuratha tatsaikadale guruḥ punaḥ punaḥ sthānaṃ | (5.334 $\frac{1}{2}$)

"Given the serial number of the unknown form, if it is (can be) halved, (write) **L**, if one has to be added in order to be halved, **G**. Place (syllables) repeatedly (in this manner) (until the requisite number of syllables have been written down.)"

Note that this is merely a rewording of his verse 5.334.

3.2.6 Jayadeva

naṣṭe yāvatithaṃ syādvṛttaṃ saṃsthāpayettamevāṅkaṃ |
ardhenāvachindyātsamaṃ hi kṛtvetaratsaikaṃ || (8.4)
evaṃ hi kriyamāṇe saike gurvakṣarāṇi labhyante |
itaratra laghūnyevaṃ pranaṣṭamutpādayedvṛttaṃ || (8.5)

"In the case of *naṣṭaṃ,* for a given serial number, establish the corresponding form. Halve the number after making it even by adding one if necessary. While doing this, put down **G**s when one is added, **L**s otherwise. Thus is obtained the missing (unknown) form."

3.2.7 Jayakīrti

pṛcchakavṛttamitāṅkaṃ dalayellaghu samadale likhedguru viṣame |
saikatvācchandomiti janayediti naṣṭavṛttarūpakamaṅkāt || (8.7)

"When asked for the form corresponding to a given serial number, (repeatedly) halve the number, writing down **L** if the number is even. In the case of odd number, add one (before halving) and write **G**. Generate the missing (unknown) form corresponding to the given serial number in this way."

3.2.8 Kedāra

naṣṭasya yo bhavedaṅkastasyārdhe'rdhe same ca laḥ |
viṣame caikamādhāya tadardhe'rdhe gururbhavet || (6.3)

"When given the serial number of a missing (form), whenever halving even, (write) **L** and whenever halving odd after adding one, (write) **G**."

3.2.9 Hemacandra

naṣṭāṅkasya dale laḥ saikasya gaḥ || (8.4)

"(Repeatedly) halving the serial number of a missing form, **L** (if the number is even), **G** if (halving) after adding one."

naṣṭāṅke gaṇairhṛte śeṣasaṅkhyo gaṇo deyo
rāśiśeṣe labdhaṃ saikaṃ || (8.5)

(*Naṣṭaṃ* in the case of *gaṇachandaḥ*) "Given the serial number of a missing form, repeatedly divide by (the number of forms of its individual) *gaṇa*s (feet), and write down the form of the *gaṇa* whose serial number is given by the remainder. (If the remainder is zero, it is considered to be equivalent to the divisor.) (If the remainder is not zero), add one (to the quotient)."

3.3 *UDDIṢṬAṂ*

3.3.1 Piṅgala

pratiloma-gaṇam dviḥ l ādyam (8.26): "opposite direction times two **L** first."

Proceeding from right to left (and with initial value one), starting at first **L** (successively) double. (here, *ganam* should be *gunam*.)

tataḥ gi ekaṃ jahyāt (8.27): "(However,) there, if **G** (is encountered), subtract one (after doubling)."

Example: **LGLG**

Initially, n=1.
start at the last L: **LG<u>L</u>G,** get n = 2
The next letter is **G**: **L<u>G</u>LG,** Get n = 2x2-1=3.
The next letter is **L**: **<u>L</u>GLG,** Get n = 2x3=6.
Done

3.3.2 Bharata

antyād dviguṇitādrupād dvidvirekam gurorbhavet |
dviguṇāñca laghoḥ kṛtvā saṃkhyāṃ piṇḍena yojayet || (121)

"From the end, (starting from) one multiplied by two, repeatedly doubled, remove one from **G**s (doubling)."

"From multiplication by two, obtaining (numbers associated with) **L**, calculate the (serial) number by aggregating."

The verse clearly describes *Uddiṣṭaṃ*. *Abhinavagupta* treats this as single algorithm, but the two lines clearly describe two different versions. The first line is a somewhat simplified version of *Piṅgala*'s algorithm except that *bhavet* does not fit. Alsdorf's version has the word *haret* instead which make sense. The second line may be interpreted as an abbreviated alternative algorithm, described more fully by *Mahāvira* (see below.) In fact, instead of translating the verse as given, *Abhinavagupta* merely substitutes the full algorithm as given by *Mahāvira, Jayadeva* and *Jayakīrti* and it runs as follows: (starting at the beginning of the verse), multiply one by two and then repeatedly multiply by two, delete the numbers associated with **G**s in the verse, add numbers associated with **L**s, finally, extend the sum (by one). (In *Bharata*'s version, the last step is missing.) For example, consider the combination **LGLG**. We get the sequence 1,2,4,8. Throw out 2 and 8, add 1 and 4: we get 5. Increase 5 by 1 to get 6 which is the index of **LGLG**.

evaṃ vinyasya vṛttānāṃ naṣtaoddiṣtavibhāgatah |
gurulaghvakṣarānīha sarvachandassu darśayet || (129)

"By putting down here **GL** syllables of forms in the case of all meters by appropriate means, *Naṣtaṃ* or *Uddiṣtaṃ*, exhibit (them)."

This is perhaps to suggest an alternate way to list all the forms of a meter as later described by *Mahāvīra* (see below.) Simply apply *naṣtaṃ* to all the indices serially.

3.3.3 Janāśraya

uddiṣṭaṃ (6.12) "*uddiṣṭaṃ*"

uddiṣṭamidānīṃ vakṣyate | "Now, let *uddiṣṭaṃ* be told."

dviguṇaṃ dviguṇaṃ vardhayetpratilomataḥ | (6.13)
kasyacicchandaso yatkiñcidvṛttaṃ vinyasyedaṃ vṛttaṃ katamadityukte tasya vṛttasyā-ntyākṣarādārabhya dvi catvāryaṣtau ṣodaśa iti pratilomato dviguṇaṃ dviguṇaṃ vardhayet |

"Increase by repeatedly multiplying by two in reverse manner (from the end to the beginning.) Put down some form of a meter. When asked after writing down some form of a meter, which form (in the enumeration) it is, beginning at the last syllable, increase (one), (to) two, four, eight, etc. by repeatedly multiplying by two proceeding in reverse order."

ekahānirbhe | (6.14) "Subtract one in case of **G**."

evaṃ pratilomato vardhayato gurau vardhite ekena hāniḥ kāryā | *evaṃ kṛtvā yathālabdha-saṃkhyāvaśādvaktavyamiti* |

"Going in reverse order in this way, in case of the long syllable, after increasing (by doubling), subtract one. Having done this, announce the number thus obtained (as the serial number of the form)."

ayamekastūpadeśaḥ (6.15) "Here (is) one more instruction (lesson)."

tatra sarveṣāṃ chandasāṃ vṛtteṣu vidhivat prastariteṣu tatrādyamardhaṃ gurvantaṃ bhavati | *aparamardhaṃ laghvantaṃ* | *tatra laghvante uddiṣṭe vṛtte uttarārdhe vṛttamiti jñātvā tasyottarā-rdhavṛttasyāntyākṣarāt prabhṛti dviguṇaṃ vardhite pūrvavadekahānīṃ kṛtvā āchando'kṣaraparisamāpteḥ pūrvārdhavṛttāni ca vaktavyānīti* | *ayaṃ tṛtīyasya kramaḥ* |

"(*Janāśraya* now attempts to explain the algorithm.) In all forms of a meter, when listed according to the algorithm, there, the first half has **G** at the end. The other half (has) **L** at the end. Therefore, when an **L**-ending form is indicated, realizing it as a form in the second half, beginning from the last syllable of the form from the second half, multiplying by two, (and) in case of **G**, subtracting one after doubling as before, until the end of (all) the syllables of the meter. The forms of the first half to be spoken of similarly."

3.3.4 Virahāṅka

antaṃ sparśaṃ gṛhītvā dviguṇāddviguṇeṣu sutanu varṅeṣu |
ekaikaṃ camareṣu muñcoddiṣṭe chāte || (6.34)

"O slender, O slender-waisted, when performing *uddiṣṭaṃ*, starting from the last **L**, repeatedly double for each syllable, subtracting one if the syllable is a **G**."

(Verses 6.36-6.40 deal with *naṣṭaṃ* and *uddiṣṭaṃ* of *gaṇachandaḥ*. See *Hemacandra* below.)

3.3.5 Mahāvīra

rūpāddviguṇottaratastūddiṣṭe lāṅkasaṃyutiḥ saikā || (5.335)

"To find the serial number of the indicated form, write down the geometric series starting with one with common ratio equal to 2 (above the syllables of the form). Add one to the total of numbers above **L**s in the form."

3.3.6 Jayadeva

uddiṣṭaṃ katithamidaṃ vṛttaṃ saṃsthāpayedupari tasya |
sthānadviguṇānaṅkānekādīnakṣarakramaśaḥ || (8.6)
ye santyupari laghūni teṣāṃ tairmiśritaistu yo rāśiḥ |
bhavati gataistāvadbhiḥ prastāravidhau tu tadvṛttaṃ || (8.7)

"What is the serial number of this form? Place above each of its location, in the order of the syllables, double numbers starting with one. (This generates the sequence, 1,2,4, . . .) Add all the numbers that are above **L**s. The number next to the total (that is, total +1) is the serial number of that form in the enumeration."

3.3.7 Jayakīrti

rūpitavṛttapratigalamekādidviguṇitāḥ syurupari tadaṅkān |
lagatānsaikānyuktvā tāvatithaṃ vṛttamiti vadetprastāre || (8.8)

"Above each **G** and **L** in a given form, let there be numbers (obtained by) repeatedly doubling the intial 1. The sum of numbers in place of **L**s with one added is the (location of) the form in the enumeration."

3.3.8 *Kedāra*

uddiṣṭaṃ dviguṇānādyāduparyaṅkānsamālikhet |
laghusthā ye ca tatrāṅkāstaiḥ saikairmiśritairbhavet || (6.4)

"Starting from the beginning (of the form), write successively double numbers above (syllables of) the entire form. The sum of those numbers which are above **L**s togetherwith one is the *uddiṣṭaṃ* (serial number)."

3.3.9 *Hemacandra*

uddiṣṭe'ntyāllāddvirgekaṃ tyajet || (8.6)

"To find the serial number of a given form, starting with the last **L** in the form, (going in reverse order), successively double (the initial one), subtracting one when **G** (is encounterd)."

ādyamantyena hataṃ vyadhastanaṃ || (8.7)

"(This verse deals with *gaṇachandaḥ*.) Starting from the end, successively multiply (the number of forms of each *gaṇa*), subtracting (after each multiplication) the number of forms of the *gaṇa* following (the given form of the gaṇa in the list of all forms of the *gaṇa*)."

3.4 *LAGAKRIYĀ*

3.4.1 *Piṅgala*

As mentioned in the Introduction, *Halāyudha* interprets the repetition of the *sūtra* "*pare pūrṅaṃiti*" as an instruction for the construction of *meru-prastāra*. This seems to me very far-fetched. (See the section on *Saṅkhyā* below.) The following *sūtra* quoted by Weber occurs in the *Yajur* recension of the *Chandaḥśāstra*, but not in its *Ṛk* recension nor in *Halāyudha*'s version.

ekottarakramaśaḥ | pūrvapṛktā lasaṃkhyā ||

"Increasing by one, step-by-step, augmented by the next, L-count."

This occurs just before the *sūtra* (8.24) in *Halāyudha*. As discussed in Section 2.4, Weber cannot make any sense out of this. The *sūtra* is most likely a cryptic *lagakriyā*. It has the elements of the algorithm given more fully *Bharata* below. For a fuller comparison, see Section 2.4.

3.4.2 Bharata

ekādhikāṃ tathā saṃkhyāṃ chandaso viniveśya tu |
yāvat pūrñantu pūrveña pūrayeduttaraṃ gañaṃ || (124)
evaṃ kṛtvā tu sarveṣāṃ pareṣāṃ pūrvapūrañaṃ |
kramānnaidhanam ekaikaṃ pratilomaṃ visarjayet || (126)
sarveṣāṃ chandasāmevaṃ laghvakṣaraviniścayam |
jānīta samavṛattānāṃ saṃkhyāṃ saṃkṣepatastathā || (127)

"Put down (a sequence, repeatedly) increased by one upto to the number (of syllables) of the meter.
Also, add the next number to the previous sum until finished.
Also after thus doing (the process of) addition of the next, (that is, formation of partial sums) of all the further (sequences),
Remove one by one, in reverse order, the terminal (number) successively.
Of all meters with (pre)determined (number of) short syllables
Thus know concisely the number of *sama* forms"

(The first word in the texts of Regnaud and Nagar is *ekādikāṃ* (numbers beginning with one) instead of *ekādhikāṃ* (numbers increasing by one) which is what Alsdorf has.

3.4.3 Janāśraya

laghuparīkṣā (6.16) "Investigation of short syllables".

laghuparīkṣedānīṃ vakṣyate | "Now the investigation of short syllables will be spoken of".

ekaikavṛddhamācchandasam (6.17) "Increasing repeatedly by one until the end of the meter."

yasya kasyacicchandasa ekalaghuvṛttānāmanyeṣāṃ ca pramāṇaṃ jijñāsamāne ekaikavṛddhānya-kṣarasthā(na)ni didṛkṣitaṃ chando'kṣarapramāni kramāt nyasyāni | evaṃ vinyasya pūrvaṃ pūrvaṃ parayutamavināśyāntyamapāsya pūrvamakṣarasthānaṃ pareṇākṣarasthānena yuktaṃ kartavyam | avināśy-āntyamakṣrasthānaṃ pūrvamevāpāsyānyatra nyasyet | tataḥ punaḥ punaḥ pūrvaṃ parayutaṃ kṛtvānyatra nyastasya parataḥ parato nyasitavyam | antyamekamavaśiṣṭaṃ bhavati | tacca teṣāṃ parato nyasitavyam | evaṃ vinyasya prathamakṣarasthānamavekṣya tadupalabdhasaṃkhyāvaśādekalaghuvṛttānīyantīti vadet | evaṃ tṛtīyādīnavekṣya trilaghuvṛttādīni brūyādityayamanyaḥ kramaḥ |

"When it is desired to know the number of forms with one **L** or others (forms with some other number of **L**s) belonging to some meter class, put down sequentially (numbers) increasing repeatedly by one, in a number of locations equal to the number of syllables in the meter. Repeatedly, add next number to the previous (sum), keeping (these numbers, except) discarding the last, (that is), combine the (content of) the location of the next syllable with (that of) the location of the previous syllable. Write elsewhere the undestroyed (saved) numbers, having removed the last number. Without destroying the location of the last syllable, discarding the

previous, put it down elsewhere. Then, put down further and further (columns of numbers) by adding again and again next to the previous and writing them elsewhere. Finally, (only the number) one is left. That too is to be written after those (earlier numbers). Written thus, announce that so many forms with one **L** according to the number obtained by observing the first location. Similarly, observing the third (location), speak of forms with 3 **L**s and so on.
(Now), here is another algorithm".

The remaining text in this section is unclear. It might be an algorithm to list the forms with a specified number of **L**s. (For example, *Varāhamihira* has such an algorithm: *Bṛhatsaṃhitā*, *Adhyāya* 76, Verse 22.) Here is the untranslated text.

sabindvādyaṃ tato dvidviḥ sabinduḥ (6.18)
sabindu adyamakṣarasthānaṃ kuryāt | pañc dviguṇadviguṇitāni didṛkṣitaṃ chandokṣara-pramāṇāni kāryāni | evaṃ kṛtvā ekalaghuvṛttādididṛkṣāyāṃ satyāṃ tadodhorūpaṃ nayet |

kramād dvirnayet (6.19)
teṣāṃ tathā nyastānāmakṣarasthānāmadhastāddidṛkṣitalaghuvṛttaprmāṇāni rūpāṇi vinyasya kramānnayet | ekaṃ dve trīṇīti gaṇayannādyādārabhyāntādevaṃ nītvā yathālabdhasaṃkhyā-vaśādekādilaghuvṛttādīni vaktavyānyetāvantītyanena sveṣu rūpeṣu vinyasteṣvantyaṃ yathā sthitasthānamekaikamiti dve ityuktamakṣarasthānaṃ nayediti | tāni punarvṛttānīdaṃ dve iti jijñāsāyāṃ satyāṃ yuktirūpāṇi vinyasya sarūpaṃ bindunā vadet | saha rūpeṇa vartata iti sarūpaṃ bindunā sārdhaṃ sarūpaṃ vigaṇayya yathālabdhasaṃkhyāvaśādidaṃ cedidaṃ ceti vaktavyaṃ | te te laghavasteṣu kasmin kasmin sthāne sthitā iti cet –

sthānāni tānyeva (6.20)
teṣāṃ laghūnāṃ sthānāni tānyeva bhavanti | etaduktaṃ bhavati – yeṣu yeṣvakṣarasthāneṣu rūpāṇyavasthitāni tānyeva sthānāni vṛtteṣu laghūnāmiti | iyamaparā laghuparīkṣā | uktā ekādaśasya vṛttasya laghavaḥ kiyanta ityukta pūrvavadakṣarasthānāni nyasya tāvat yāvadbhiḥ sabindubhiḥ sthānairekādaśasaṃkhyā paripūrṇā bhavati tāvantastasyaikādaśasya vṛttasya laghavo bhavanti | evaṃ viśeṣāṇāmapi jñeyam| atra tatsthānāni tānyeva bhavanti |

3.4.4 Virahāṅka

pramukhente ca ekaikaṃ tathaiva madhya ekamabhyadhikaṃ |
prathamādārabhya vardhante sarvāṇkāḥ || (6.7)

ekaikena bhajyate uparisthitaṃ tathaiva |
paripāṭyā muñcaikaikaṃ sūciprastāre || (6.8)

tatpiṇḍyatāṃ nipuṇaṃ yāvad dvitīyamapyāgataṃ sthānaṃ |
prastārapātagaṇanā laghukriyā labhyate saṃkhyā || (6.9)

(*Sūci prastāra*) "Put down the numeral 1, in the beginning, the end and in between (as many as the number of syllables in the meter) and one more. Increase all the numbers starting with the first (as follows.)"

“One-by-one, add the number above (to the partial sum). In the *Sūci* prastāra, successively leave out (the last number) one-by-one.”

“The accumulation is complete when the second place is reached (until the number to be left out of addition is in the second place.) *Laghukriyā* number is obtained by carrying out the algorithm.” (Velankar interprets the last line of 6.9 to mean that all the numbers of *Laghukriyā* are to be added up to obtain the total number of forms of a meter.)

iha koṣṭakayordvayorvardhate adhaḥsthitaṃ krameṇaiva |
pramukhānte ekaikaṃ tataśca dvau trayaścatvāraḥ || (6.10)

uparisthitāṅkena vardhate'dhaḥsthitaṃ krameṇaiva |
merau bhavati gaṇanā sūcyā eṣa anuharati || (6.11)

sāgaravarṇe'ṅkau dvaveva gurū madhyamasthāne |
samare punareka eva merau tathaiva sūcyāṃ || (6.12)

(*Meru*) “Two cells (rectangles) in a place, successively increase (the number of cells) below them. In the first and last cell (enter) numeral 1 in (rows) 2, 3, 4 (etc).”

“Step-by-step, in (each) cell below, (place) the sum of the numbers in the (two) cells above. The calculation of the Sūci prastāra is (re)created in the (table called) *meru* (named after the mythical mountain). This (procedure) imitates (it.)”

“In the case of odd number of syllables, there are two large(st) numbers in the middle, moreover, in the case of even number of syllables, there is only one (such) in the *meru*, just as in *Sūci prastāra*.”

3.4.5 Mahāvīra

(Mathematician *Mahāvīra* uses the modern formula for calculating combinatorial coefficients here instead of following the procedure used by the prosodists. This formula already appears in *Pāṭigaṇita* of *Śridhara* in the section on combinations.)

ekādyekottarataḥ padamūrdhvādharyataḥ kramotkramaśaḥ |
sthāpya pratilomaghnaṃ pratilomaghnena bhājitaṃ sāraṃ |
syāllaghugurukriyeyaṃ saṅkhyā dviguṇaikavarjitā sādhvā || (5.336 $\frac{1}{2}$)

“(Write down) the arithmetic sequence starting with one and common difference equal to one upto the number of syllables in the meter above, and in reverse order below (the same sequence). Product of the numbers (first, first two, first three, etc.) (of the sequence) in reverse order divided by the product of the corresponding numbers (of the sequence) in forward order is the *laghukriyā* Total number of forms multiplied by 2 minus one is *adhvā*.”

For example, with n=6, we have the sequences 1, 2, 3, 4, 5, 6 and 6, 5, 4, 3, 2, 1. This gives us the successive combinatorial coefficients

$$\frac{6}{1} = 6, \quad \frac{6 \bullet 5}{1 \bullet 2} = 15, \quad \frac{6 \bullet 5 \bullet 4}{1 \bullet 2 \bullet 3} = 20, \text{ etc}$$

3.4.6 Jayadeva

vṛttākṣarāṇi yāvantyekenādhikatarāṇi tāvanti |
ūrdhvakrameṇa rūpāṇyādau vinyasya teṣāṃ tu || (8.8)

ādyaṃ kṣipeddvitīye dve ca tṛtīye'tha tānyapi caturthe |
evaṃ yāvadupāntyaṃ kuryāttvevaṃ hi bhūyo'pi || (8.9)

yadadho bhavantyupāntyāttatprabhṛti punaḥ kramānnivartante |
ekadvitrilaghūni prathamād guruṇo bhavantyeva || (8.10)

"Put down one above the other, number of 1's equal to the number of syllables in the meter plus one. Add first (top most) to the second, then second to the third, then the third to the fourth. Do this until you reach the penultimate place. Repeat this again and again. (At the end of the process), the punultimate number and the numbers below indicate (the number of forms with) one **L**, two **L**s, three **L**s (and so on), from the first (place), (obtain) the number of forms with all **G**s."

3.4.7 Jayakīrti

chandovarṇānekādhikarūpānutkramānnidhāyā(+dhastā)t |
tattaduparyupari tatha kśipediti punaḥ punarjahannekaikaṃ (8.9**)**
ādyanta (nte?) sarvalage ekādilaghūni madhyavṛttānyeṣu | (8.9 $\frac{1}{2}$**)**

"Having put down number of 1's, as many as the number of syllables in the meter plus one. Add repeatedly the number above, (repeat this) again and again, leaving out one (last) number one-by-one. The first number is the number of forms consisting of **L**s and the others are (the numbers of) forms with one (two, three) etc. **L**s."

3.4.8 Kedāra

varṇānvṛattabhavānsaikānauttarādharyataḥ sthitān |
ekādikramaśaścaitānuparyupari nikṣipet || (6.5)
upāntyato nivarteta tyajannekaikamūrdhvataḥ |
uparyādyād gurorevamekadvyādilaghukriyā || (6.6)

"Place one above the other as many numeral ones as the number of syllables in the meter plus one. Beginning with the first number, successively add to the sum the number above upto the penultimate number. Leave out (numbers) one-by-one from the top. From the top, from the first (number), (the number of forms) with (only) **G**'s, (then) the number of forms with one, two, etc. **L**s."

3.4.9 Hemacandra

varṇasamānekakān saikānuparyupari kṣipet|
muktvāntyaṃ sarvaikādigalakriyā || (8.8)

"(Write down as many) numeral 1's as the number of syllables plus one. Repeatedly add the next number above, leaving out the last. This is *lagakriyā* for all (**L**s) beginning with one."

ādyabhedānadho'dho nyasya parairhatvāgre kṣipet || (8.9)

"(Now the extension to *gaṇachandaḥ*.) Write down the numbers corresponding to the types of forms of the first *gaṇa*, one below the other. Multiply them with those of the second and add the resulting columns. (continue the process.)" (See Alsdorf for a translation of related *Hemachandra*'s commentary on this.)

3.5 SAṄKHYĀ (S_n)

3.5.1 Piṅgala

dvirardhe (8.28) "two in case of half."

If n can be halved, write "twice".

rūpe śūnyam (8.29) "In case one (must be subtracted in order to halve), 'zero'."

dviḥ śūnye (8.30) "(going in reverse order), twice if zero'."

tāvadardhe tadguṇitam (8.31) "In case where the number can be halved, multiply by itself (that is, square the result.)"

Example: $n = 6$
First construct the second column in the table below. Second, going back up, construct the third column:

n=6	twice	$(2\cdot 2^2)^2 = 64$
n=3	zero	$2\cdot 2^2 = 8$
n=2	twice	$2^2 = 4$
n=1	zero	2

Thus, total number S_6 of possible forms of length 6 is 64.

dvird(v?)ūnaṃ tadantānām (8.32)

"twice two-less that (quantity) replaces (the sequence of counts) ending (with the current count)."

Twice s_n minus 2 equals sum of the series ending with S_n.

$$2S_n - 2 = S_1 + S_2 + \cdots + S_n$$

pare pūrṅaṃ (8.34) “next full”

pare pūrṅaṃiti (8.35) “next full, and so on.”

That is, subsequent S_n’s are full double of the previous, without subtraction of 2.

$$S_{n+1} = 2S_n.$$

Sūtra (8.35) is a repetition of *sūtra* (8.34). Its interpretation by *Halāyudha* as an instruction to construct the modern-day Pascal’s triangle to implement “*lagakriyā*” makes no sense. On the other hand, repeating a word or a phrase is a common usage in Sanskrit to indicate repetition of an action just described, namely, repeated doublings to determine the successive S_n‘s.

To calculate *saṇkhyā* of *ardhasama* and *viṣama* forms, *Piṅgala* gives the following algorithm earlier in his composition. It is tempting to conjecture that his divide-and-conquer algorithm (8.28-31) is inspired by the algorithm below.

samamardhasamaṃ viṣamaṃ ca (5.2)

“*sama* (equal), *ardhasama* (half-equal), *viṣama* (unequal).”

samaṃ tavatkṛtvaḥ kṛtamardhasamaṃ (5.3)

“By multiplying (the number of) *sama* by itself (one obtains) *ardhasama*.”

Sūtra 5.5 below clarifies that one has to subtract the number of *sama* from this.

viṣamaṃ ca (5.4) “and (similarly, the number of) *viṣama*.”

raśyūnaṃ (5.5) “Subtract the quantity”

That is, subtract the quantity from its square. Therefore, the number of *ardhasama* is the square of *sama* minus the *sama*. Similarly, the number of *viṣama* is the twice-squared *sama* minus the square of *sama*.

Piṅgala has a single formula dealing with *Mātrāchandaḥ*:

sā g yena na samā lāṃ gla iti (4.53)

“The (the number of) **G** (is equal to that quantity) by which the number of syllables is unequal from the number of *mātrā*s.” The formula is: #**G** = #mātrā - #syllable.

3.5.2 Bharata

ādyaṃ sarvaguru jñeyaṃ vṛttantu samasaṃjñitam |
kośaṃ tu sarvalaghvantyaṃ miśrarupāñi sarvataḥ || (122)

Abhinavagupta interprets this verse as a brief description of *Lagakriyā* and calculation of *Saṅkhyā* (total number of forms of a meter) therefrom. Literal translation of the verse suggests that this is merely a simple characterization of *prastāra*s. The verse reads as follows, with "*kośaṃ*" replaced by "*kośe*" as in Alsdorf's version: "In the tabulation, *sama* forms are known to have all **G**s at the beginning, all **L**s at the end, mixed syllables everywhere (else)." The verse seems to a prelude to the next verse dealing with *ardhasama* and *viṣama* forms.

vṛttānāntu samānāṃ saṃkhyāṃ saṃyojya tāvatīm |
rāśyūnamardhaviṣamāṃ samāsādabhinirdiśet || (123)

"(Replacing ardhaviṣamāṃ by ardhasamāṃ, the verse reads:) The number of *sama* forms, after multiplying by as much (squaring it), less the (original) quantity, precisely specifies the number of *ardhasama*."

samānāṃ viṣamāñāṃ ca saṁguñayya tathā sphuṭam |
rāśyūnamabhijānīyadviṣamāñāṃ samāsataḥ || (125)

"(*viṣamāñāṃ* in the first line should clearly read *ardhasamānāṃ*. With that change, the verse reads:) (The number) of *viṣama* forms is known precisely by what becomes evident after subtracting the original quantity from the multiplication (by itself) of the sum of the number of *sama* forms and the number of *ardhasama* forms."

3.5.3 Janāśraya

saṃkhyā (6.21) "*saṃkhyā* (total number of forms)"

saṃkhyedānīṃ vakṣyate | yasya kasyacicchandasaḥ samavṛttasaṃkhyādididṛakṣāyāṃ satyāṃ tasya pādākṣarāṇi vinyasya tataḥ -

"Let us now speak about *saṃkhyā*. When it is desired to know *saṃkhyā* of *sama* forms of a meter, after putting down the (number of) syllables," -

bhārdhahṛte (6.22) "**G** when divided into half"

vinyastānāṃ pādākṣarāṇāmardhaṃ hṛtvā gurunyāsaḥ punaḥ punarevakāryaḥ | ardhe punarhniyamāṇe yadi viṣamatā syāt –

"dividing the number of syllables into half, having put down **G**, repeat the same again and again. If, when trying to divide into half, oddness occurs, -"

ho samamapāsyaikam (6.23) "**L** (when made) even (by) subtracting one."

ekamapāsya pūrvanyastasyādho laghuṃ nyasaivaṃ sarvāṇyapanayet | eṣa saṃkhyāgarbhaḥ |

"Subtracting one, putting down **L** below what was put down before, (and thus) divide all (numbers.) This is the essence (meaning) of *saṃkhyā*."

bhe dviḥ (6.24) "In case of **G**, twice."

laghau laghau dvidvi kuryāt | tāvatā bhe guṇayet vardhyedityarthaḥ | uktamevārthaṃ nirūpayiṣyāmaḥ |

"Whenever (you have) **L**, double. In the case of **G**, multiply that quantity by itself, thus increase (the number.) This said, we will illustrate its meaning."

The rest of the commentary on this sūtra shows that this procedure yields the number of *sama* forms of the *gāyatri* meter which consists of 4 feet of 6 syllables each as 64.

dyūnaṃ tadantānām (6.25) "two-less replaces ending with that."

taddvirityanuvartate tad gāyatrī samavṛttasaṃkhyāpramāṇaṃ dviguṇīkṛtaṃ ca dvābhyāṃ hīnaṃ tadantānāṃ gāyatryantānāṃ ṣaṇṇāṃ samavṛttasaṃkhyāpramāṇaṃ bhavati | tattu ṣadviṃśatyadhi- kaśatamevaṃ viśeṣāṇāmapi jñeyam | samavṛttādhigame brūmaḥ - samavṛtto vargamūlo gādhv-ardhasamaḥ | samatulena guṇito varga ityucyate | rāgiṇaḥ samavargastasya mūle gāścedardh-
samavṛttādhikaraṇaṃ bhavati | samavargastu catvāri sahasrāṇi ṣaṇṇavatiśca sa tu mūlonaścatvāri sahasraṇi dvātriṃśadadhikāni |

"follow (the rule) 'twice that'. (The *sūtra* obviously refers to *Piṅgala*'s original *sūtra*.) Thus, after doubling the *saṃkhyā* of *sama* forms of the *gāyatri* meter and subtracting two from it, the (total) number of (*sama* forms of the first) six meters ending with the *gāyatri* is obtained. Thus, 126 is known as *saṃkhyā* in this particular case. Having obtained (the number) for *sama* forms, we now say: square of (number of) *sama* forms; by subtracting the original, (get) (number of) *ardhasama* forms. Square is defined as multiplying by the equal (the same) quantity. The formula for *ardhasama* is the square of *sama* (forms) of *rāgin* (*gāyatri*) (less the original (quantity). (In the case of the *gāyatri* meter), square of *sama* is 4096, that less the original is 4032."

ubhayavargo viṣamaḥ (6.26) "twice square *viṣama*"

mūlona iti vartate | ubhayeṣāṃ samānāṃ vṛttānāṃ vargaḥ sa tu mūlonaḥ viṣama vṛttapramāṇaṃ bhavati | ubhayavargaḥ kiyāniti cet pratiloma ekaḥ ṣaṭ saptātha sapta dve caikameva ṣadvargaḥ samārdhasamyorgāyatrīya kathayedbudhaḥ mūlaṃ tu catvāri sahasrāṇi ṣaṇṇavatiśca tena hīna ubhayavargaḥ |
ekaṃ ṣaṭ sapta saptātha trīṇyekaṃ dve ca śūnyakam |
pramāṇaṃ viṣamāṇāṃ tu gāyatryā lakṣayedbudhaḥ ||
evamanyeṣāṃ chandasāṃ jñeyam |

"(The rule) 'Less square root' (still) applies. Subtraction of the square root from the twice-squred *sama* yields the number of *viṣama* forms. If asked how much is twice square, in reverse order: one six seven and two more sevens one and also six (16777216), the square of *sama* and *ardhasama* of *gāyatri,* the experts say; the square root is 4096, twice-square less that (square root): one six seven seven again three one two and zero (16773120); the expers recognize (this) as the the measure (number) of *viṣama* forms of the *gāyatri* meter.
(The number of forms) of other meters are found similarly."

āsamūhairyathāsvamalpabhedānniṣpannānāṃ vaitālīyādīnāṃ jātiślokānāṃ lāghave mātrā yena pramāṇena yāvatāṅgenākṣarebhyo'dhikā bhavatīti |

"In *mātrā*meters, *vaitālīya* etc, constructed by means feet of 4 *mātrā*s, namely, *gaṅgā*, *kurute*, *vibhāti*, *sātava*, *nacarati*, and also feet consisting of 6, 7 and 8 *mātrā*s, number of *mātrā*s equals the amount by which twice the number of syllables exceeds the number of short syllables."

3.5.4 Virahāṅka

chando yāvatsaṃkhyaṃ sthāpayitvā sthāpaya tasya pādāṅkaṃ |
anenaiva guṇitenārdhena bhavanti gurulaghavaḥ || (6.41)

"After putting down the number (of forms of the foot) of a meter, put down the number of syllables in the foot. The product of the two numbers divided by 2 yields the total number of **G**s and **L**s."

kṛtvā varṇagaṇanaṃ mātrā bhavanti yā adhikāḥ |
te guravaḥ śeṣāḥ punarlaghavaḥ sarvāsu jātiṣu || (6.45)

"The number of **G**s equals the excess of *mātrā*s over the number of syllables. Moreover, the number of **L**s the remainder among all the syllables."

#**G** = #Mātrās - #Syllables; #**L** = #Syllables - #**G**

antimavarṇāddviguṇaṃ varṇe varṇe [ca] dviguṇaṃ ku[ruta] |
pādākṣaraparimāṇaṃ saṃkhyāyā eṣa nirdeśaḥ || (6.46)

"Starting from the last syllable, double (the initial one) for each syllable upto the number of syllables in the meter. This shows the total count (of all the forms of a meter.)"

evaṃ ca varṇavṛtte mātrāvṛttānāmanyathā bhavati |
dvau dvau pūrvavikalpau yā mīlayitvā jāyate saṃkhyā |
sā uttaramātrāṇāṃ saṃkhyāyā eṣa ni(r)deśaḥ || (6.49)

"Thus for the case of syallable-(based)-forms, but it is different for the *mātrā*-(based)-forms. The count (of all possible forms of a *mātrā*meter) is obtained by adding (the counts of) permutations of the two previous (*mātrā*meters). This is the way to the count (of total permuations) of succeeding *mātrā*s (*mātrā*meters)."

Thus, we get the formula $S_n = S_{n-1} + S_{n-2}$ which generates what is now known as the Fibonacci sequence.

3.5.5 Mahāvīra

samadalaviṣamasvarūpaṃ dviguṇaṃ vargīkṛtaṃ ca padasaṃkhyā || (5.333)

"Halve if the number is even; add one and halve if odd; (Going in reverse order) square the number (in the first case) and double it (in the second case). (Thus is obtained) the number of (possible forms of) a foot (with specified number of syllables)."

3.5.6 Jayadeva

eṣveva piṇḍiteṣu ca saṃkhyā prastāraviracitā bhavati |
uddiṣṭavidhānāṅkaiḥ saikairmiśrībhavantyathavā || (8.11)

"The sum of all the numbers (obtained *laghukriyā*) is the total number of forms constructed by the *prastāra*. (Alternatively) add all the numbers written down during *uddiṣṭaṃ* (above each syllable) and add one."

That is, *saṃkhyā* = $1+2+2^2+\cdots+2^{n-1}+1$.

3.5.7 Jayakīrti

piṇḍīkṛteṡu saṃkhyā saikoddiṣṭāṅkapiṅditā vā saṃkhyā || (8.10)

"When numbers used in *uddiṣṭaṃ* are added and the sum is increased by one, the total number of forms (of a meter) is obtained."

chando'kṣare samadale śūnyaṃ nyasya viṣame tathā rūpaṃ |
rūpe taddviguṇaṃ khe vargaḥ samavṛttasaṃkhyā syāt || (8.11)

"(Alternatively) if the number of syllables in the meter is even, halve it and put down zero. If number is odd, put down one (and halve after subtracting one.) (Continue until one is reached.) (Going backwards) if (the number that was put down) is one, multiply by 2; if it is a zero, square the number. The total number of *sama* forms is (thus) obtained."

jñātasamavṛttasaṃkhyā tattadguṇato'rdhasamakasaṃkhyā mūlāt |
tattadguṇātsamūlārdhasamamiterviṣamavṛttamitirapamūlā || (8.12)

"The known count of *sama* forms multiplied by itself equals the count of *ardhasama* forms including the original (*sama* forms). That *ardhasama*, combined with the original (*sama* forms) (and) multiplied by itself, is the count of *viṣama* forms after the original (that is, the quantity that was squared) is subtracted."

jātyaṃhricatuṣke pratigaṇaṃ kṣipettatra saṃbhavadgaṇasaṃkhyāṃ |
guṇa(+ye)danyonyaṃ tatsaṃkhyā syātsarvajātisaṃkhyeti matā || (8.14)

"In the case of *mātrā*meters with four feet, write down the number of possible forms for each foot. Multiply these numbers. The product is the approved *saṃkhyā* of *mātrā*meters."

jātermātrāpiṇde svākṣararahite kṛte sthitā guravaḥ syuḥ |
gururahite'kṣarasaṃkhyā laghurahite'rdhīkṛte tu gurusaṃkhyā syāt || (8.16)

"In the case of *mātrā*meters, number of *mātrā*s minus the number of syllables is (the number of) **G**s. The number of *mātrā*s minus the number of **G**s (is) the number of syllables. The number of *mātrā*s minus the number of **L**s, halved, gives the number of **G**s."

3.5.8 Kedāra

laghukriyāṅkasaṃdohe bhavetsaṃkhyā vimiśrite |
uddiṣṭāṅkasamāhāraḥ saiko vā janayedimāṃ || (6.7)

"*Saṃkhyā* is obtained by adding together the numbers obtatained by *lagakriyā*. Alternatively, it may be obtained by adding one to the sum of *uddiṣṭaṃ* numbers."

3.5.9 Hemacandra

te piṇditāḥ saṃkhyāḥ || (8.10)

"*Saṃkhyā* is the sum of those (obtained by *lagakriyā*)."

varṇasamadvikahatiḥ samasya || (8.11)

"(Alternatively) (The number) of *sama* forms (is) the product of as many 2's as there are number of syllables."

te dviguṇā dvihīnāḥ sarve || (8.12)

"That number multiplied by 2 (and then) reduced by 2 (is) all." (That is, the number of all *sama* forms of meters with syllables from 1 upto the number of syllables of the present meter.)

samakṛtī rāśyūnā ardhasamasya || (8.12)

"Square of the *sama* forms minus the original quantity is the number of *ardhasama* forms."

tatkṛtirviṣamasya || (8.14)

"Square of that (minus the original quantity) is the number of *viṣama* forms."

vikalpahatirmātrāvṛttānāṃ || (8.15)

"The number of forms of a *mātrā*meter (is) the product of number of forms (of its individual feet)."

aṅkāntyopāntyayogaḥ pare pare mātrāṇāṃ (8.16)

"The sum of the last and the one before the last is the number of forms of the next *mātrā*-foot."

This is the formula $S_n=S_{n-1}+S_{n-2}$ given by *Virahāṅka* earlier.

3.6 ADHVAYOGAḤ

3.6.1 Piṅgala

Halāyudha's version of *Piṅgala*'s *Chandaḥśāstra* does not include this 6th *pratyaya*, but Weber quotes the following *sūtra* from the *Yajur* recension of the *Chandaḥśāstra* (occurring just before the *sūtra* corresponding to *sūtra* (8.24) in *Halāyudha*):

aṅgulapṛthuhastadaṇḍakrośāḥ | *yojanaṃ ity adhvā* ||

"*aṅgula* (finger), *pṛthu* (palm), *hasta* (hand), *daṇḍa* (staff), *krośā* (shout), *yojana*, thus space."

Weber also quote the following sūtra from the Ṛk recension occurring between (8.32) and (8.33).

ekone'dhvā

"when one (is) subtracted, space"

Clearly, both refer to *adhvayogaḥ*. The *Yajur* version lists units of measurements and ends with "thus space" without telling us how to compute it. The *Ṛk* version gives the actual formula, "Space when one is subtracted". That is, to obtain the amount of space required to write down all the forms of a particular meter, double the total number of forms and subtract one. The formula allows the writer to leave space between successive forms.

3.6.2 Bharata

Bharata omits *adhvayogaḥ*.

3.6.3 Janāśraya

adhvā |6.27) "Space."

adhavā idānīṃ vakṣyate | "Now speak about space"

vṛttaṃ dvirekonam | (6.28) "(The number of) forms twice less one."

vṛttāni dviguṇīkṛtāni ekena hīnāni adhvapramāṇaṃ bhavati | sarveṣāṃ chandasāṃ tatra gāyatryā adhvayogaṃ darśayiṣyāmaḥ | asyāḥ sarvāṇi vṛttāni dviguṇīkṛtāni –
tāni trṇyatha pañcapi pañca catvāri tattvataḥ |
catvāri trṇi cāpyevaṃ pramitānyaṅgulāni vai ||

"The measure of space is twice the (number of) forms less one. Among all meters, we will illustrate the space calculation of the space for gāyatri. Multiply by two (the number of) all of its forms (and subtract one): Indeed, they exactly measure two three's five five four four three and one (33554431.)"

aṅgulamaṣtau yavā dvādaśāṃgulāni vitastihasto dvau hastau kiṣkudhanuḥ dhanuḥ sahasre dve krośaścatvāraḥ krośā yojanam |

"Two Eights (16) *yava*s (grains of barley) (equals) an *aṅgula* (finger), twelve *aṅgula*s measure a *hasta* (hand), two *hasta*s (equal) a *kiṣkudhanu* (forearm-bow), *dhanu* when two thousand (equals) *krośa* (shout), four *krośa*s (equal) a *yojana*."

(In the next paragraph in coverting 33,554,431 *aṅgula*s converted into *yojana*s, a different conversion table is implicitly assumed: 1 *hasta* = 24 *aṅgula*s, 1 *dhanu* = 4 *hasta*s, 1 *yojana* = 8000 *dhanu*s.)

tanyaṅgulāni hastasya saṃkhayā trayodaśalakṣāṇi mannuv(?)daśaśataṃ caikāsaptāṃgulāni | dhanuḥ saṃkhayā trīṇi lakṣāṇi catvāryayutāni nava sahasrāṇi ca pañcaśatāni pañcaviṃśatiśca | viṃśatiśca(?) hastaścaikaḥ yojanasaṃkhayā trayaścatvāriṃśadyojanārdhayojanaśca dhanuṣāṃ sahasrāṇi pañcaviṃśatyadhikāni pañca śatāni ca saptāṅgulādhiko hastaścaika iti sabhāhā ca yenā-
kṣarebhyo'dhikā sa khalu gururbhavati |

(Dividing) by the number of *aṅgula*s in a *hasta*, (33,554,431 *aṅgula*s equal) 1,398,101 (*hasta*s) and 7 *aṅgula*s. (1,398,101 divided) by the number of *hasta*s in a *dhanu* (equals) 349,525 (*dhanu*s) and 1 *hasta*. (Dividing) by the number of *dhanu*s in a *yojana*, (the final result is) 43½ *yojana*s, 1525 *dhanu*s, 1 hasta and 7 *aṅgula*s. Thus, the length of that which is constructed with the two syllables, **G** and **L**, is indeed obtained."

A hasta of 24 *aṅgula*s must measure at least a foot. Assuming that, the space requirement of 1,398,101 *hasta*s amounts to about 265 miles!

3.6.4 Virahāṅka

caturaṅgulaśca rāmastribhiḥ rāmaiḥ jānīhi vitastiṃ |
dvau vitastī hastaścaturhasto dhanurdharastathā || (6.56)
dve eva dhanuḥsahasre krośasya bhavati parimāṇaṃ |

krośā aṣṭau tathaiva yojanasaṃkhyā vinirdiṣṭā || (6.57)

"4 *Aṅgula*s (fingers) = 1 *Rāma*; 3 *Rāma*s = 1 *Vitasti*; 2 *Vitasti*s = 1 *Hasta* (hand); 4 *Hasta*s = 1 *Dhanu* (bow); 2000 *Dhanu*s = 1 *Krośa*; 8 *Krośa*s = 1 *Yojana*."

ekāṅgulaṃ ca ruṇaddhi camaraḥ sparśopyaṇgulaṃ caiva |
camarasparśāntarāle ekamevāṅgulaṃ bhavati || (6.58)

"A **G** covers one *aṅgula* and an **L** also covers one *aṅgula*. The space between (syllables) **G, L** is one *aṅgula*.

3.6.5 Mahāvīra

Mahāvīra includes this in his verse (5.336 $\frac{1}{2}$) dealing with *lagakriyā* where he also calculates the total number of forms of a meter.

dviguṇaikavarjitā sādhvā || (5.336 $\frac{1}{2}$)

"Total number of forms multiplied by 2 minus one is *adhvā*."

3.6.6 Jayadeva

dvābhyāṃ samāhatā saṃkhyā rūpeṇaikena varjitā |
chinnavṛttāṅgulavyāptiradhvayogaḥ prakīrtitaḥ || (8.12)

"The amount of space measured in *aṅgula* (fingers) occupied by all the (written) forms including the space between successive lines is twice the *saṃkhyā* (of a given meter) minus one. This is known as *adhvayoga*."

3.6.7 Jayakīrti

chandaḥsaṃkhyā hatā dvābhyāmekarūpavivarjitā |
chinnavṛttāṅgulavyāptiradhvayogo bhavediti || (8.17)

"The amount of space measured in *aṅgula* (fingers) occupied by all the (written) forms including the space between successive lines is twice the *saṃkhyā* (of a given meter) minus one. This is *adhvayoga*."

māṇḍavyapiṅgalajanāśrayasaitavākhya-
śrīpādapūjyajayadevabudhādikānāṃ |
chandāṃsi vīkṣya vividhānapi satprayogān
chando'nuśāsanamidaṃ jayakīrtinoktaṃ || (8.19)

"After studying variously employed meters of Revered *Jayadeva*, the first among the wise, expounded by *Māṇḍavya, Piṅgala, Janāśraya*, (and) *Saitava*, this (is) *Chando'nuśāsana* narrated by *Jayakīrti*."

3.6.8 Kedāra

saṃkhyaiva dviguṇaikonā sadbhiradhvā prakīrtitaḥ |
vṛttasyāṅgulikīṃ vyāptinadhaḥ kuryāttathāṅgulaṃ || (6.8)

"*saṃkhyā* multiplied by two less one is known as the space (required by) all (of *prastāra*). The width of one form is one *aṅguli* (finger) and below it make (space of) another *aṅguli* (allowing space between successive forms.)"

3.6.9 Hemacandra

dvighnānekādhvayogaḥ || (8.17)

"twice (*saṃkhyā*) minus one (is) the space (required to write down the entire *prastāra*)."

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