Bhāskarācārya′s Līlāvatī

(Part II Covering the Topics)

Geometry, First Degree Indeterminate Equation and Permutations

Translated and Edited

By

A. B. Padmanabha Rao

Bhāskarācārya′s Līlāvatī

(Part II Covering the Topics)

Geometry, First Degree Indeterminate Equation and Permutations

A Translation from Sanskrit into English with Sanskrit Text
and Roman Transliteration

With Word by Word Meaning in the English Text Order
Of 138 Ślokas and Gaṇeśadaivajña′s
The Buddhivilāsinī Commentary.

Translated and Edited
By
A. B. Padmanabha Rao

सत्यं परं धीमहि

Chinmaya International Foundation Shodha Sansthan
Adi Sankara Nilayam, Veliyanad
Ernakulam, Kerala
2014

Chinmaya Research Series: 11
ISBN: 978-93-80864-18-1

Bhāskarācārya′s Līlāvatī - Part II
(Geometry, First Degree Indeterminate Equation and Permutations)
Translated and Edited By A. B. Padmanabha Rao

Published with the financial assistance of Rashtriya Sanskrit Sansthan, Ministry of Human Resource Development,
Govt. of India, New Delhi

Publisher : Chinmaya International Foundation Shodha Sansthan
(Recognised by Rashtriya Sanskrit Sansthan and Mahatma Gandhi University),
Adi Sankara Nilayam, Veliyanad,
Ernakulam District - 682 313, Kerala, India

Tel/Fax: 91-484-2747104
E-mail: director@chinfo.org
Website: www.chinfo.org

First Edition: 2013, 300 copies
Price: ₹380

Table of Contents

8
Permutations

Combinatorial Mathematics is used in fields as diverse as Prosody, Ventilation of Halls, Architecture, Medicine, Khaṇḍameru (Pascal's triangle), and Rasaśāstra (Science of flavours) among many others, which due to fear of lengthy elaboration is not given here - Bhāskara[7].

[Mathematics is an Interdisciplinary Subject- Editor]

8.1 Permutations and their Sum

अथाङ्कपाशः - *athāṅkapāśaḥ*

Now about a string of digits.

अथ गणितपाशे निर्दिष्टाङ्कैः सङ्ख्याया विभेदे करणसूत्रं वृत्तम्।

atha gaṇitapāśe nirdiṣṭāṅkaiḥ saṅkhyāyā vibhede karaṇasūtraṃ vṛttam.

Now, a stanza to find the permutations of given digits in a number.

Śloka 261

स्थानान्तमेकादिचयाङ्कघातः सङ्ख्याविभेदा नियतैः स्युरङ्कैः।
भक्तोऽङ्कमित्याङ्कसमासनिघ्नः स्थानेषु युक्तो मितिसंयुतिः स्यात्।।२६१।।

sthānāntamekādicayāṅkaghātaḥ saṅkhyāvibhedā niyataiḥ syuraṅkaiḥ,
bhakto'ṅkamityāṅkasamāsanighnaḥ sthāneṣu yukto mitisaṃyutiḥ syāt..261..

एकादि–चयाङ्क–घातः	स्थानान्तम्	स्युः
ekādi-cayāṅka-ghātaḥ	*sthānāntam*	*syuḥ*
The product of 1, 2, 3, ... upto the last digit	of a given number N	shall be
सङ्ख्याविभेदाः	नियतैः अङ्कैः	(सः घातः)
saṅkhyāvibhedāḥ	*niyataiḥ aṅkaiḥ*	*(saḥ ghātaḥ)*
number of permutations P	of given n digits.	(That product P)

निघ्नः	अङ्कसमास–	भक्त अङ्कमित्या	युक्ताः
nighnaḥ	*aṅkasamāsa-*	*bhakta aṅkamityā*	*yuktāḥ*
multiplied by	sum of digits s	divided by number of digits n,	the result S' being filled
स्थानेषु	स्यात्	मितिसंयुतिः	
sthāneṣu	*syāt*	*mitisamyutiḥ*	
in places of digits	shall be	sum S of all the number of permutations.	

Explanation

The number of permutations of string of digits $a_1a_2a_3...a_n = 1.2.3....n =$ P say. The śloka says:

$S' = \mathrm{P}s/n$ ... (i) where

$s = a_1 + a_2 + ... + a_n$ and the sum S of all the permutations $a_1a_2...a_n + a_2a_1...a_n + ...$ (for $n = 6$ say) is given by

10^5 ... 10^2 10 1

S' ... S' S' S'. Thus generalising

S $= S'(1\ ...\ 1\ 1\ 1) = S'(1 + 10 + 10^2 + ... + 10^{n-1})$. ... (ii)

In modern notation:

P $= 1.2.3\ ..\ n = {}^nP_n = n!$ ... (i)

[*Buddhivilāsinī* [1] explains:

एकश्चेदङ्कस्तदाऽस्यैक एव भेदः ... ... भेदतुल्या एवाऽऽवर्तन्ते

ekaścedaṅkastadā'syaika eva bhedaḥ ... ...bhedatulyā evā"vartante

Here is the gist:

If the string is one digit number the number of permutations is 1.

If it is 2 digit one, the second place may be filled in 2 ways (left or right of 1) i.e. the number of permutations = 1.2

[1] *Līlāvatī*[7], pp. 276-277

If it is 3 digit one, the third place may be filled in 3 ways (empty spaces between and beyond 1 and 2) i.e. the number of permutations = 1.2.3
Similarly for more and more digits.]

[**Ed.** *Buddhivilāsinī* fills the empty spaces between and beyond the occupied digits, starting with one, two, three digits and so on, from the given number of digits.

The modern method, on the other hand, starts with filling n empty spaces by given digits, one by one, as follows:
First space can be filled by a digit, in n different ways. Having filled one, the second can be filled, by a second digit, in $n-1$ ways, giving a total of $n(n-1)$ ways. Similarly for third place it is $n(n-1)(n-2)$ ways, and so on.]

[*Buddhivilāsinī* [2] explains by considering the 3 digit number $a_1a_2a_3$
The 6 permutations are

$a_1a_2a_3$

$a_1a_3a_2$

$a_2a_1a_3$

$a_2a_3a_1$

$a_3a_1a_2$

$a_3a_2a_1$

It further comments that the presentation given above looks like a lock of abundance of braided hair (rajjupāśā), hence the name aṅkapāśaḥ is given to this chapter.

Thus in each of the 3 columns, the digits a_1, a_2, a_3 are repeated 1(2)(=6/3) times. Similarly for 4 digit number $a_1a_2a_3a_4$ there are 24 permutations of 4 digits. Thus in each of the 4 columns, the digits a_1, a_2, a_3, a_4 are repeated 1.2.3(=24/4) times.

[2] *Līlāvatī*[7], pp.277 :
तद्यथा– स्थानद्वयाङ्कप्रस्तार ... भवतीत्युपपन्नम्
tadyathā sthānadvayāṅkaprastāra ... bhavatītyupapannam

Therefore sum of each column

$S' = (a_1 + a_2 + a_3 + a_4)24/4$. Hence sum S of all 24 permutations is given by $S = S'S'S'S' = S'(1111) = S'(1 + 10 + 100 + 1000)$, treating S' as having a place value as if it is a digit. $S'S'S'S'$ stands for a 4 'digit' number each 'digit' standing for the number S'.

Similarly for n digit number $a_1a_2a_3$ ... a_n the sum of all 1.2.3 ... n permutations S is given by

S = $S'S'...S'(111 ... 1)$, where $S' = s(1.2.3 ... n)/n$ and $s = a_1+a_2+a_3+...a_n$]

In modern notation

S = $sn!(111...11)/n = sn!(10^{n-1} + ... + 10^2 + 10 + 1)/n$ i.e.,

S = $sn!(10^n - 1)/9n$. ... (ii)

अत्रोद्देशकः–

atroddeśakaḥ

Here is an example.

Śloka 262

द्विकाष्टकाभ्यां त्रिनवाष्टकैर्वा निरन्तरं द्वयादिनवावसानैः।

सङ्ख्याविभेदाः कति सम्भवन्ति तत्सङ्ख्यकैक्यानि पृथग्वदाऽऽशु।।२६२।।

dvikāṣṭakābhyāṃ trinavāṣṭakairvā nirantaraṃ dvayādinavāvasānaiḥ,

saṅkhyāvibhedāḥ kati sambhavanti tatsaṅkhyakaikyāni pṛthagvadā"śu..262

पृथग्वदाऽऽशु	कति	सङ्ख्यविभेदाः
pṛthagvadā ´´su	*kati*	*saṅkhyāvibhedāḥ*
Tell quickly and separately	how many	permutations of
द्विकाष्टकाभ्यां वा	त्रिनवाष्टकैर्वा च	अवसानैः
dvikāṣṭakābhyām vā	*trinavāṣṭakairvā ca*	*avasānaiḥ*
2, 8 or	3, 9, 8 and	also of digits
निरन्तरं	द्वयादिनव	तथा
nirantaram	*dvayādinava*	*tathā*
continuously	from 2 to 9,	and likewise,

तत्सङ्ख्यैक्यानि	सम्भवन्ति
tatsaṅkhyakaikyāni	*sambhavanti*
the sum of the permutations,	which arise.

Here are 3 examples of sets of digits.

(i) (2, 8) (ii) (3, 9, 8) (iii) (2, 3, 4, 5, 6, 7, 8, 9)

Answers:

(i) Number of digits = 2, their sum =10 and

the number of their permutations = 1(2) = 2

Sum of digits in each place = 1(2)(2 + 8)/2 = 10

Sum of permutations is

Tens	Units	
10	10	= 10(11) = 110

(ii) No. of digits = 3, their sum = 20 and

the number of their permutations = 1(2)(3) = 6

Sum of digits in each place = 1(2)(3)(20)/3 = 40

Sum of permutations is

Hundreds	Tens	Units	
40	40	40	or
44	4	0	i.e, 4440

Buddhivilāsinī gives the following steps:

Hundreds	Tens	Units	
40			
4	0		
	4	0	
44	4	0	i.e, 4440.

(iii) Number of digits = 8, their sum = 44 and

the number of their permutations = 1.2.3.4.5.6.7.8 = 40320

Sum of digits in each place = 40320(44)/8 = 221760

8.2 Applications

उदाहरणम्- Example.

Śloka 263

पाशाङ्कुशाहिडमरूककपालशूलैः खट्वाङ्गशक्तिशरचापयुतैर्भवन्ति।
अन्योऽन्यहस्तकलितैः कति मूर्तिभेदाः शम्भोर्हरेरिव गदारिसरोजशङ्खैः।।२६३।।

pāśāṅkuśāhiḍamarūkakapālaśūlaiḥ khaṭvāṅgaśaktiśaracāpayutairbhavanti,
anyo'nyahastakalitaiḥ kati mūrtibhedāḥ śambhorharereiva gadārisaroja-śaṅkhaiḥ..273..

कति *kati* How many	मूर्तिभेदाः *mūrtibhedāḥ* different idols	भवन्ति *bhavanti* can be formed	अन्योन्य– *anyonya-* by exchanging
हस्तकलितैः *hastakalitaiḥ* weapons in hands	पाश– *pāśa-* noose,	अङ्कुश– *aṅkuśa-* goad,	अहिडमरूक– *ahidamarūka-* snake, drum
कपाल– *kapāla-* skull,	शूलैः *śūlaiḥ* trident,	खट्वाङ्ग– *khaṭvāṅga-* staff with skull at the top	शक्ति– *śakti-* miscile,
शर– *śara-* arrow	चापयुतैः *cāpayutaiḥ* with bow	शम्भोः *śambhoḥ* of *Śiva*,	गदा– *gadā-* mase
अरि– *ari-* disc	सरोज– *saroja-* lotus	शङ्खैः *śaṅkhaiḥ* and conch	हरेरिव *harereiva* like those of Hari's.

There are 10 weapons in the 10 hands of *Śiva*.

Number of permutations of His idols = 1.2.3.4.5.6.7.8.9.10 = 36,28,800.

There are 4 weapons in the 4 hands of Hari.

The number of permutations of His weapons = 1.2.3.4 = 24*

* It is interesting to note that the number 24 corresponds to the 24 syllables of Gāyatri mantra and the 24 names of Hari in the ācamana viz., starting with Keśava, Nārāyaṇa, Mādhava, ..., Hari and Kṛṣṇa. Reciting this mantra had been a part of the daily Hindu religious ritual(Sandhyāvandanam) in ancient India.

8.2.1 Special case of Repeated Digits

अथ विशेषकरणसूत्रं वृत्तम्।

atha viśeṣakaraṇasūtraṃ vṛttam.

Now an algorithm for the special case in one *śloka*.

Śloka 264

यावत्स्थानेषु तुल्याङ्कास्तद्भेदैस्तु पृथक्कृतैः।
प्राग्भेदा विहृता भेदास्तत्सङ्ख्यैक्यं च पूर्ववत्।।२६४।।

yāvatsthāneṣu tulyāṅkāstadbhedaistu pṛthakkṛtaiḥ,
prāgbhedā vihṛtā bhedāstatsaṅkhyaikyaṃ ca pūrvavat..264..

प्राग्भेदाः *prāgbhedāḥ* The previous number of permutations	विहृताः पृथक्कृतैः *vihṛtāḥ pṛthakkṛtaiḥ* divided separately	तद्भेदैः *tadbhedaiḥ* by the number of those permutations
तुल्याङ्काः *tulyāṅkāḥ* of the repeated digits	यावत्स्थानेषु (स्युः) *yāvatsthāneṣu (syuḥ)* in as many places,	भेदाः *bhedāḥ* shall be the number of permutations,
पूर्ववत्सङ्ख्यैक्यं च *pūrvavat saṅkhyaikyaṃ ca* and sum of the permutations is obtained as before.		

[*Buddhivilāsinī*[3] explains:

[3] *Līlāvatī*[7], p.280.

अत्रोपपत्तिः – तुल्याङ्कानामन्योन्य ... सम्यग्स्युरिति

atropapattiḥ - tulyāṇkānāmanyonya ... samyagsyuriti

Permutations arise in the case of repeated digits. It is not proper to count them. Hence the previously obtained permutations have to be divided by the permutations of the repeated digits.]

Explanation

Let number of places = n. Sum of digits = s.

Number of places containing a repeated digit = n_1.

Number of places containing another repeated digit = n_2.

Then the number of permutations = $\mathrm{P}/(P_1 P_2)$. ... ... (i)

The sum of the digits in each column, $S' = sP/(nP_1P_2)$.

where $\mathrm{P} = 1.2.3 \ldots n$, $\mathrm{P}_1 = 1.2.3 \ldots n_1$, $\mathrm{P}_2 = 1.2.3 \ldots n_2$.

Sum of all permutations = $S'(111 \ldots 1) = S'(10^{n-1} + .. + 10 + 1)$. (ii)

It can be extended to any number of sets of repetitions.

अत्रोद्देशकः – *atroddeśakaḥ*

Now, an example.

Śloka 265

द्विद्व्येकभूपरिमितैः कति सङ्ख्यकाः स्यु–
स्तासां युतिश्च गणकाऽऽशु मम प्रचक्ष्व।
अम्भोधिकुम्भिशरभूतशरैस्तथाङ्कैः
चेदङ्कपाशमितियुक्तिविशारदोऽसि।।२६५।।

dvidvyekabhūparimitaiḥ kati saṅkhyakāḥ syu-
stāsāṃ yutiśca gaṇakā"śu mama pracakṣva,
ambhodhikumbhiśarabhūtaśaraistathāṅkaiḥ
cedaṅkapāśamitiyuktiviśārado'si..265..

हे गणक	प्रचक्ष्व मम	आशु
he gaṇaka	*pracakṣva mama*	*āśu*
O! mathematician	tell me	quickly,

कति सङ्ख्यकाः	द्विद्व्येकभूपरिमितैः	तासां युतिः
kati saṅkhyakāḥ	*dvidvyekabhūparimitaiḥ*	*tāsām yutiḥ*
how many permutations	with two 2s and two 1s, are there	(and also) their sum,

तथाङ्कैः	अम्भोधि–	कुम्भि–	शर–भूत–	शरैः
tathāṅkaiḥ	*ambhodhi-*	*kumbhi-*	*śarabhūta-*	*śaraiḥ*
and from digits	4	8	5 5	and 5,

चेत् असि युक्तिविशारदः	अङ्कपाशमिति	तासां युतिश्च
cet asi yuktiviśāradaḥ	*aṅkapāśamiti*	*tāsām yutiśca* .
if you are skilled expert	in permutations	and in summing them.

[**Ed.** Even though they can be solved by using the formulae, Bhāskarācārya by way of explanation, actually displays all the permutations in each case, and demonstrates the problem. In the case of first example he displays all the permutations of the digits 2, 2, 1 and 1 as follows:

2211, 2121, 2112, 1212, 1221 and 1122, whose sum is 9999. Similarly he demonstrates the solution of the second example by actual enumeration of the permutations. 48555, 84555, 54855, ...

Example (i)
The digits are 2, 2, 1, 1, $n = 4,\ n_1 = 2,\ n_2 = 2,\ s = 2 + 2 + 1 + 1 = 6$
$P_1 = 2,\ P_2 = 2$, P = 1.2.3.4 = 24.
The number of permutations = $P/(P_1P_2) = 6$, from (i)
Sum of the digits in each column = $s\mathrm{P}/(P_1P_2n) = 6(24)/(2)(2)(4) = 9$
The sum of all permutations = 9(1111) = 9999, from (ii)

Example (ii)
The digits are 4, 8, 5, 5, 5 n = 5, $n_1 = 3, P_1 = 6$, P = 120, s = 27
Number of permutations = P/P_1 = 20
Sum of digits in each column = 20(27)/5 = 108
The sum of permutations = 108(11111) = 11,99,988]

8.3 Permutations with no Digits Repeated

अनियताङ्कैरतुल्यैश्च विभेदे करणसूत्रं वृत्तार्धम्।

aniyatāṅkairatulyaiśca vibhede karaṇasūtram vṛttārdham.

A rule for permutations of unknown number, with no digits repeated in half a stanza.

Śloka 266

स्थानान्तमेकापचितान्तिमाङ्कघातोऽसमाङ्कैश्च मितिप्रभेदाः।।२६६।।

sthānāntamekāpacitāntimāṅkaghāto'samāṅkaiśca mitiprabhedāḥ..266..

एकापचितान्तिमाङ्क–घातः	स्थानान्तम्
ekāpacitāntimāṅka-ghātaḥ	*sthānāntam*
The product of 9 and digits 8, 7, ..	till the last place (of the number)

स्युः	मितिप्रभेदाः	असमाङ्कैः
syuḥ	*mitiprabhedāḥ*	*asamāṅkaiḥ*
shall be	number of permutations	with distinct digits.

[**Ed.** Here the actual digits (of an n-digit number), which are unequal, are not given. The number of permutations P of n distinct digits is

P = 9.(9 - 1)(9 - 2) ... (9 - (n -1)).

For 1 digit the permutations are 9.

For 2 digit number, the 2^{nd} digit can be any of the remaining 8 digits.

Hence the permutations are 9(8). Similarly for 3 digits it will be 9(8)7 etc.

Note the maximum number of places is 9.]

उदाहरणम् – Example

Śloka 267

स्थानषट्कस्थितैरङ्कैरन्योन्यं खेन वर्जितैः।
कति सङ्ख्याविभेदाः स्युर्यदि वेत्सि निगद्यताम्।।२६७।।

sthānaṣaṭkasthitairaṅkairanyonyaṃ khena varjitaiḥ,
kati saṅkhyāvibhedāḥ syuryadi vetsi nigadyatām..267..

यदि वेत्सि	निगद्यताम्	कति	अन्योन्यं
yadi vetsi	*nigadyatām*	*kati*	*anyonyaṃ*
If you know	(then) say	how many	mutual

विभेदाः	खेन वर्जितैः	सङ्ख्या स्युः	स्थान–षट्क–स्थितैः
vibhedāḥ	*khena varjitaiḥ*	*saṅkhyā syuḥ*	*sthāna-ṣaṭka-sthitaiḥ*
permutations	of non-zero	numbers are there	in 6 places.

[*Buddhivilāsinī*[4] explains the logic by considering two-digit numbers. There are 90 two-digit numbers. Nine of them end with 0, such as 10,20,...,90 and another nine have repeated digits like 11,22,...,99. Thus the total permutations are 90-9(2) = 9(8). Similarly the case of 3 digit number is discussed.]

[**Ed.** Here $n = 6$. Therefore number of permutaions P is given by
P = 9(9 - 1)(9 - 2)(9 - 3)(9 -4)(9 - 5) = 9.8.7.6.5.4 = 60,480]

8.4 Permutations Given the Sum of Digits

Śloka 268 - 269

निरेकमङ्कैक्यमिदं निरेकस्थानान्तमेकापचितं विभक्तम्।
रूपादिभिस्तन्निहतैः समाः स्युः सङ्ख्याविभेदा नियतेऽङ्कयोगे।।
नवान्वितस्थानकसङ्ख्यकाया ऊनेऽङ्कयोगे कथितं तु वेद्यम्।
सङ्क्षिप्तमुक्तं पृथुताभयेन नान्तोऽस्ति यस्माद्गणितार्णवस्य।।२६९।।

nirekamaṅkaikyamidaṃ nirekasthānāntamekāpacitaṃ vibhaktam,
rūpādibhistannihataiḥ samāḥ syuḥ saṅkhyāvibhedā niyate'ṅkayoge.
navānvitasthānakasaṅkhyakāyā ūne'ṅkayoge kathitaṃ tu vedyam,
saṅkṣiptamuktaṃ pṛthutābhayena nānto'sti yasmādgaṇitārṇavasya..269..

अङ्कैक्यम् निरेकम्	इदम् स्थाप्यम्	निरेकस्थानान्तम्
aṅkaikyam nirekam	*idam sthāpyam*	*nirekasthānāntam*
The sum s of digits be reduced by 1.	This is to be kept,	in places up to the penultimate one,
एकापचितम्	विभक्तम्	रूपादिभिः
ekāpacitam	*vibhaktam*	*rūpādibhiḥ*
reducing each time by 1	and divided	(respectively) by 1, 2, 3, ...

[4] *Līlāvatī*[7], pp. 281-282.

तन्निहतैः	समाः स्युः	सङ्ख्याविभेदाः
tannihataiḥ	*samāḥ syuḥ*	*saṅkhyāvibhedāḥ*
these, when multiplied,	shall be equal to	number of permutations
नियते अङ्कयोगे	कथितं तु	वेद्यम्
niyate aṅkayoge	*kathitam tu*	*vedyam*
in the given sum of digits.	What is said is however,	understood to be (applicable),
अङ्कयोगे ऊने (सति)	नवान्वित–स्थानक–सङ्ख्यकाया	
aṅkayoge ūne (sati)	*navānvita-sthānaka-saṅkhyakāyā*	
while the sum of given digits is less than	9 added to number of digits, n.	
(एतत्) उक्तं	सङ्क्षिप्तम्	पृथुताभयेन
(etat) uktam	*saṅkṣiptam*	pṛthutābhayena
This is said	in brief	lest it should be too long,
यस्मात् नास्ति अन्तः	गणितार्णवस्य	
yasmāt nāsti antaḥ	*gaṇitārṇavasya*	
for there is no end	to the ocean of mathematics.	

Explanation

Given the sum s of n digits to find the number of P permutations:

If $s < n + 9$ the formula is

$$P = \frac{(s-1)(s-2)...(s-n+1)}{1.2.3...(n-1)}$$

$$= (s-1)_{C(n-1)}$$, in modern notation.

The case $s >= n + 9$ is not dealt with, as said above, let it become too long.

उदाहरणम् - Example

Śloka 270

पञ्चस्थानस्थितैरङ्कैर्यद्यद्योगस्त्रयोदश।
कति भेदा भवेत्सङ्ख्या यदि वेत्सि निगद्यताम्।।२७०।।

pañcasthānasthitairaṅkairyadyadyogastrayodaśa,
kati bhedā bhavetsaṅkhyā yadi vetsi nigadyatām..270..

यदि	वेत्सि (तर्हि)	निगद्यताम्	कति	सङ्ख्याभेदाः
yadi	*vetsi (tarhi)*	*nigadyatām*	*kati*	*saṅkhyābhedāḥ*
If you	know (then)	say	how many	permutations

तेषाम्	पञ्चस्थानस्थितैः अङ्कैः	यद्यद्योगः त्रयोदश	भवेत्
teṣām	*pañcasthānasthitaiḥ aṅkaiḥ*	*yadyadyogaḥ trayodaśa*	*bhavet*
of those	5 digit numbers	each of whose sum is 13,	will be there?

This is an example on Theory of Partitions[19]
The number of places $n = 5$. Their sum $s = 13$.
The number P of permutations = 12.11.10.9/1.2.3.4 = 495.

[*Buddhivilāsinī* actually enumerates all permutations, as shown below:
There are in all 18 combinations of 5 different types, enumerated below, the sum of digits in each type is 13.

If one digit is repeated, others being distinct
(i) 2 times, it can only be 1, 2 or 3 of the type
(11236, 11245, 22351, 33142.),
with number of permutations **60** each, giving total permutations
= 4(60) = *240*
Here **60** = (1.2.3.4.5)/1.2
(ii) 3 times it can only be 1 or 2 of the type
(11164, 11137, 11146, 22243, 22261),
with number of permutations **20** each, giving
total permutations = 5(20) = *100*
Note **20** = (1.2.3.4.5)/1.2.3.
(iii) 4 times it can only be 1, 2 or 3 of the type (11119, 22225, 33331),
with number of permutations **5** each, giving
total permutations = 3(5) = *15.* As before **5** = (1.2.3.4.5)/1.2.3.4.

If two digits are repeated, others being distinct

(iv) 2 times they can only be 1,2,3 or 4 of the type (11227, 11335, 11443, 22441), with number of permutations **30** each, giving total permutations = 4(30) = *120*

Here again **30** = (1.2.3.4.5)/((1.2).(1.2)).

(v) 2 and 3 times respectively they can only be 5, 2 twice, and 3 and 1 thrice of the type (11155, 33322), with number of permutations **10** *each*, giving total permutations = 2(10) = *20*.

Finally we note that **10** = [(1.2.3.4.5)/(1.2)(1.2.3)]

Total of these permutations being *240 + 100 + 15 + 120 + 20 = 495*.]

Śloka 271

न गुणो न हरो न कृतिर्न घनः पृष्टस्तथाऽपि दुष्टानाम्।
गर्वितगणकबटूनां स्यात्पातोऽवश्यमङ्कपाशेऽस्मिन्।।२७१।।

na guṇo na haro na kṛtirna ghanaḥ pṛṣṭastathā'pi duṣṭānām.
garvitagaṇakabaṭūnāṃ syātpāto'vaśyamaṅkapāśe'smin..271..

न गुणः	न हरः	न कृतिः	न घनः
na guṇaḥ	*na haraḥ*	*na kṛtiḥ*	*na ghanaḥ*
Neither multiplication,	nor division,	nor squares	nor cubes

पृष्टस्तथाऽपि	पातः	दुष्टानाम्	गर्वितगणकबटूनां
pṛṣṭastathā'api	*pātaḥ*	*duṣṭānām*	*garvitagaṇakabaṭūnāṃ*
are asked, still	the fall of	evil and	proud mathematicians

अवश्यम् स्यात्	अस्मिन्	अङ्कपाशः
avaśyam syāt	*asmin*	*aṅkapāśaḥ*
is certain	in these	(calculations of) permutations.

Buddhivilāsinī[5] suggests that Bhāskarācārya is highlighting the importance of the tough topic of permutations by saying that it can only be learnt by the intelligent ones because it cannot be learnt by arithmetic or algebraic methods using the rule of three.

[5] *Līlāvatī*[7], pp.284

[**Ed.** Though, as said before, Indian mathematics is application oriented, there are several occassions in which Mathematics for its own sake is developed. The best example of this is Piṅgala′s Chandaśśāstra. While Chandaśśāstra deals with prosody, it seems the combinatorial mathematics of prosody is developed mostly for its own sake. This is one of the best instances of development of mathematics, which happened beyond the application for which it was meant because very few combinations from combinatorial mathematics find place in the realm of poetry. Glimpses of it can be had in the paper Recursion and Combinatorial mathematics in Chandaśśāstra[1]]

8.5 Use of Pun and its Interpretations

Śloka 272

येषां सुजातिगुणवर्गविभूषिताङ्गी
शुद्धाऽखिलव्यवहृतिः खलु कण्ठसक्ता।
लीलावतीह सरसोक्तिमुदाहरन्ती
तेषां सदैव सुखसम्पदुपैति वृद्धिम्।।२७२।।

yeṣāṃ sujātiguṇavargavibhūṣitāṅgī
śuddhā'khilavyavahṛtiḥ khalu kaṇṭhasaktā,
līlāvatīha sarasoktimudāharantī
teṣāṃ sadaiva sukhasampadupaiti vṛddhim..272..

This stanza has two meanings depending upon how the words are interpreted and thus gives rise to two versions.

First version:

इह	येषाम्	कण्ठसक्ता	लीलावती
iha	*eṣam*	*kaṇṭhasaktā*	*Līlāvatī*
In this world	those by whom	is memorized	*Līlāvatī* (arithmetic)

सुजाति–गुण–वर्ग–विभूषिताङ्गी	शुद्धा–अखिल–	व्यवहृतिः
sujāti-guṇa-varga-vibhūṣitāñgī	*śuddhā-akhila*	*vyavahṛtiḥ*
adoroned with, rules of reducing	(and) complete	day to day (problems)
fractions, square, multiplication,	faultless	

उदाहरन्ती	सरसोक्तिम्	सदैव खलु उपैति	वृद्धिम्	सुखसम्पत्
udāharantī	*sarasoktim*	*sadaiva khalu upaiti*	*vṛddhim*	*sukhasampat*
illustrated with	interesting instances,	will always certainly attain	the increase in	joy and prosperity.

Second version:

Literary interpretation, being very flexible, gives rise to different shades of meanings depending on the point of view taken by the interpreter!

इह	येषाम्	कण्ठसक्ता	लीलावती
iha	*eṣām*	*kaṇṭhasaktā*	*Līlāvatī*
In this world	those by whom is	embraced	*Līlāvatī*(a lass)

सुजाति–गुण–वर्ग–विभूषिताङ्गी	शुद्धा–अखिल–	व्यवहृतिः
sujāti-guṇa-varga-vibhūṣitāñgī	*śuddhā-akhila-*	*vyavahṛtiḥ*
adorned with, good lineage* and a host of qualities,	complete and faultless	conduct,

उदाहरन्ती	सरसोक्तिम्	सदैव खलु उपैति	वृद्धिम्	सुखसम्पत्
udāharantī	*sarasoktim*	*sadaiva khalu upaiti*	*vṛiddhim*	*sukhasampat*
conversing in	a beautiful speech(language),	will always certainly attain	the increase in	joy and prosperity.

**Buddhivilāsinī*and *Līlāvatīvivaraṇa*[6]: Types of women such as Padminī etc.
शोभना जाति भोगजात्यादि ...-*śobhanā jāti bhogajātyādi ...*

Perhaps this has led some to connote increase in joy and prosperity to mean - carnal desires. [24]

[6] *Līlāvatī*[7], p.285