# Bhāskarācārya′s Līlāvatī

(Part I Covering the Topics )

Arithmetic and Algebra

**Translated and Edited**

**By**

**A. B. Padmanabha Rao**

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(Part I Covering the Topics )

Arithmetic and Algebra

*A Translation from Sanskrit into English with Sanskrit Text*
*and Roman Transliteration*

With Word by Word Meaning in the English Text Order
Of 138 Ślokas and Gaṇeśadaivajña′s
The Buddhivilāsinī Commentary.

**Translated and Edited**
**By**
**A. B. Padmanabha Rao**

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(Arithmetic and Algebra)

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# 9
# Combinations

In this chapter various combinations of selecting a part of a given group of things are considered. Permutations form a topic of *Līlāvatī* second part. Bhāskarācārya says this topic has a wider application to various fields. Here are a few examples.

1) Prosody[1](Science of metres in poetry) dealing with various combinations of long and short syllables occurring in poetic metres. The study of this science by its author has led to the binary representation of numbers, and its relation to the decimal representation.

2)Architecture and Arts including fine arts such as Music, Dance etc. which involve various combinations of structures, musical notes , beats,
the foot works etc.

3) In building constructions involving various windows in walls and ceilings for good breeze and ventilation.

4) In Medicines and cooking involving various combinations of chemicals and flavours respectively.

5) Khaṇḍa Meru (the so called Pascal′s triangle) leading to binomial coefficients and their relations and many more which are not enumerated lest the list be too long.

अथ च्छन्दश्चित्यादौ करणसूत्रं श्लोकत्रयम्––

*atha cchandaścityādau karaṇasūtraṃ ślokatrayam–*

Now combinations of metres in Prosody etc. - a rule in three ślokas.

[1] *chandasśāstra* by Piñgala, 300 B.C.

**Śloka** 112

एकाद्येकोत्तरा अङ्का व्यस्ता भाज्याः क्रमस्थितैः ।
परः पूर्वेण सङ्गुण्यस्तत्परस्तेन तेन च ।। ११२ ।।

*ekādyekottarā aṅkā vyastā bhājyāḥ kramasthitaiḥ .*
*paraḥ pūrveṇa saṅguṇyastatparastena tena ca .. 112 ..*

| | | |
|---|---|---|
| एकाद्येकोत्तरा अङ्का व्यस्ता | भाज्याः क्रमस्थितैः । | परः पूर्वेण तेन (भक्तेन*) सङ्गुण्यः |
| *ekādyekottarā aṅkā vyastā* | *bhājyāḥ kramasthitaiḥ .* | *paraḥ pūrveṇa tena (bhaktena)saṅguṇyaḥ* |
| The numbers 1, 2 etc. placed in *reverse* order | be divided by 1, 2 etc. in *this* order. | The next (quotient) be multiplied by the previous one, |

| | |
|---|---|
| तत्परस्तेन | एकद्वित्र्यादिभेदाः स्युः** |
| *tatparastena* | *ekadvitryādibhedāḥ syuḥ* |
| the next (*quotient*) by the previous (*product*). | These (results) shall be the combinations of one, two, three ...(from a group of $n$ things) |

*लब्ध्या- labdhyā(by quotient)

** From the next śloka.

The above śloka stands for the following arrangement

| Order | | | | | | |
|---|---|---|---|---|---|---|
| Reverse | $n$ | $n-1$ | $n-2$ | ... | 2 | 1 |
| Direct | 1 | 2 | 3 | ... | $n-1$ | $n$ |
| Quotient | $\frac{n}{1}$ | $\frac{(n-1)}{2}$ | $\frac{(n-2)}{3}$ | ... | $\frac{2}{(n-1)}$ | $\frac{1}{n}$ |
| Product | $\frac{n}{1}$ | $\frac{n(n-1)}{1.2}$ | $\frac{n(n-1)(n-2)}{1.2.3}$ | ... | $\frac{n(n-1)\ldots 2}{1.2\ldots(n-1)}$ | $\frac{n(n-1)\ldots 1}{1.2\ldots n}$ |
| Modern notation | $n_{C1}$ | $n_{C2}$ | $n_{C3}$ | ... | $n_{C(n-1)}$ | $n_{Cn}$ |

These are the combinations of $n$ things taken 1, 2, 3 ... etc. at a time.

[ **Ed.**

The logic of the above arrangements:

The first row represents number of way in which :

1 can be selected, and having selected 1

2 can be selected, having selected 1 2

3 can be selected and so on.

Second row represents the *orders* :

1, 2, 3, ... ,$n-2$, $n-1$, $n$ in which they *are* selected.

Third row implies that,

since the order of selection is immaterial, they are removed by respective divisions

$\frac{n}{1}, \frac{(n-1)}{2}, \frac{(n-2)}{3}, \ldots \frac{3}{(n-2)}, \frac{2}{(n-1)}, \frac{1}{n}$.

The last row represents the successive products

$\frac{n}{1}, \frac{n(n-1)}{1.2}, \frac{n(n-1)(n-2)}{1.2.3}, \ldots \frac{n(n-1),\ldots 3}{1.2\ldots(n-2)}, \frac{n(n-1)\ldots 2}{1.2\ldots(n-1)}, \frac{n(n-1)\ldots 1}{1.2\ldots n}$ which are the actual number of combinations of selecting 1, 2, 3, ... at a time from $n$ things. ]

**Śloka** 113

एकद्वित्र्यादिभेदाः स्युरिदं साधारणं स्मृतम् ।

छन्दश्चित्युत्तरे छन्दस्युपयोगोऽस्य तद्विदाम् ।। ११३ ।।

*ekadvitryādibhedāḥ syuridaṃ sādhāraṇaṃ smṛtam .*

*chandaścityuttare chandasyupayogo ´sya tadvidām .. 113 ..*

| | | |
|---|---|---|
| इदं साधारणं स्मृतम् छन्दश्चित्युत्तरे | छन्दसि उपयोगोऽस्य तद्विदाम् | मूषावहनभेदादौ* |
| *idaṃ sādhāraṇaṃ smṛtam chandaścityuttare* | *chandasi upayogo ´sya tadvidām* | *mūṣāvahanabhedādau* |
| This is generally remembered in prosodical combinations, | useful to the learned in prosody, | in finding combinations of breeze-carrying air-holes etc., |

*From the next śloka.

**Śloka** 114

मूषावहनभेदादौ खण्डमेरौ च शिल्पके ।
वैद्यके रसभेदीये तन्नोक्तं विस्तृतेर्भयात् ।। ११४ ।।
*mūṣāvahanbhedādau khaṇḍamerau ca śilpake .*
*vaidyake rasabhedīye tannoktaṃ vistṛterbhayāt .. 114 ..*

| | | |
|---|---|---|
| खण्डमेरौ च | शिल्पके। वैद्यके | तन्नोक्तं विस्तृतेर्भयात् |
| *khaṇḍamerau ca* | *śilpake. vaidyake* | *tannoktaṃ vistṛterbhayāt* |
| combinations arranged so as to form a part of a mountain⋆(Pascal's triangle), and | arts*, medicine, combinations of various tastes. | It is not said(much) lest it be too long. |

* Musical notes, beats and foot work in dance, architecture, building constructions, etc.
⋆ This is the Pascal′s triangle.

तत्र छन्दश्चित्युत्तरे किञ्चिदुदाहरणम्–
*tatra chandaścityuttare*
*kiñcidudāharaṇam-*
Some examples on combinations in prosody.

**Śloka** 115

प्रस्तारे मित्र गायत्र्याः स्युः पादे व्यक्तयः कति ।
एकादिगुरवश्चाऽऽशु कति कत्युच्यतां पृथक् ।। ११५ ।।
*prastāre mitra gāyatryāḥ syuḥ pāde vyaktayaḥ kati.ekādi-*
*guravaścā´´śu kati katyucyatāṃ pṛthak .. 115 ..*

| | | |
|---|---|---|
| मित्र उच्यताम् आशु | गायत्र्याः प्रस्तारे कति व्यक्तयः | एकादिगुरवः स्युः पादे |
| *mitra ucyatām āśu* | *gāyatryāḥ prastāre kati vyaktayaḥ* | *ekādiguravaḥ syuḥ pāde* |
| O! friend tell me quickly | in a Gayatrī metre◇ how many combinations | of one, two, etc., of long vowels are there in a line. |

| | | |
|---|---|---|
| कति च कति च | स्युः | (उच्यतां) पृथक् |
| *kati ca kati ca* | *syuḥ* | *(ucyatāṃ) pṛthak* |
| How many* and how many⋆ | will there be ? | (let it be said) separately. |

* The total in a line of *Gāyatrī*

⋆ The total in the 4 lines of the mantra.

◇ Normally it has 4 lines of 6 syllables each.

The Vedic representation of the mantra consists of 3 lines of 8 syllables each.

Bhāskarācārya gives the following solution

| | | |
|---|---|---|
| इह हि षडक्षरो | गायत्रीचरणः । | अतः षडन्तानामेकाद्यङ्कोत्तर अङ्कानां |
| *iha hi ṣaḍakṣaro* | *gāyatricaraṇaḥ.* | *ataḥ ṣaḍantānāmekā dyaṅkottara aṅkānāṃ* |
| Here indeed are 6 letters | in a line of *gāyatri.* | Hence the numbers 1, 2, ... , 6 |

व्यस्तानां क्रमस्थानां च

*vyastānāṃ kramasthānāṃ ca*

in reverse and direct orders are

न्यासः - *nyāsaḥ*

as given in the data:

६ । ५ । ४ । ३ । २ । १ ।

*6 . 5 . 4 . 3 . 2 . 1 .*

१ । २ । ३ । ४ । ५ । ६ ।

*1 . 2 . 3 . 4 . 5 . 6 .*

| | | |
|---|---|---|
| यथोक्तकरणेन | लब्धाः एकगुरु व्यक्तयः ६ । | द्विगुरवः १५ । |
| *yathokta karaṇena* | *labdhā ekaguru vyaktayaḥ, 6* | *dviguravaḥ, 15* |
| As said before are obtained | combinations of 1 long syllable, 6 | of 2 long syllables, 15 |

| | | |
|---|---|---|
| त्रिगुरवः २० । | चतुर्गुरवः १५ | पञ्चगुरवः ६ |
| *triguravaḥ, 20* | *caturguravaḥ, 15* | *pañcaguravaḥ, 6* |
| 3 long syllables, 20 | 4 long syllables, 15 | 5 long syllables, 6 |

| | | | |
|---|---|---|---|
| षड्गुरुः १ । | अथैकः सर्वलघुः १ | एवमासामैक्यं | पादव्यक्तिमितिः ६४ |
| *ṣaḍguruḥ 1* | *athaikaḥ sarvalaghuḥ 1* | *evamāsā maikyaṃ* | *pādavyaktimitiḥ 64* |
| and 6 long syllables, 1 | and one with all short syllables, 1. | Thus adding all, | the combinations in a line are 64 |

Here

$1 + 6 + 15 + 20 + 15 + 6 + 1 = 64.$

In modern notation

$1 + 6 + 15 + 20 + 15 + 6 + 1$

$= 6_{C0} + 6_{C1} + 6_{C2} + 6_{C3} + 6_{C4} + 6_{C5} + 6_{C6}$

$= (1 + 1)^6 = 2^6 = 64.$

In fact this is a particular instance of the formula

$\sum n_{Cr} = (1+1)^n = 2^n$, $r = 0, 1, 2, \ldots\ n.$

The khaṇḍameru mentioned in the above śloka represents combinations[1]. in the form of the triangle given below. This resembles Pascal's triangle.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | | 1 | | | | | | |
| | | | | 1 | | 2 | | 1 | | | | | |
| | | | 1 | | 3 | | 3 | | 1 | | | | |
| | | 1 | | 4 | | 6 | | 4 | | 1 | | | |
| | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | | |
| 1 | | 6 | | 15 | | 20 | | 15 | | 6 | | 1 | |
| ... | ... | ... | ... | ... | ... | ... | ... | | | | | | |

| | | |
|---|---|---|
| एवं चतुश्चरणाक्षरसंख्यका– न्यथोक्तं | विन्यस्यैकादिगुरुभेदा– नानीय | तान्सैकानेकीकृत्य जाता |
| *evaṃ catuścaraṇākṣara-saṃkhyakānyathoktaṃ* | *vinyasyaikādiguru-bhedanānīya* | *tānsaikānekīkṛtya jātā* |
| Thus the combinations for 4 lines obtained | by bringing the various* combinations | and adding them become |

| | | |
|---|---|---|
| गायत्रीवृत्तव्यक्तिसंख्याः १६७७७२१६ | एवमुक्थाद्युत्कृतिपर्यन्तं | छन्दसां व्यक्तिमिति र्ज्ञातिव्या |
| *gāyatrīvṛttavyakti-saṃkhyāḥ 16777216* | *evamukthādyutkṛti-paryantaṃ* | *chandasāṃ vyakti-mitirjñātavyā .* |
| the combinations of *gāyatri* Śloka $16777216 = 64^4$. | Thus from *ukthā** onwards up to *utkṛti** | the combinations in prosody should be known. |

* Various metres starting from 1 letter onwards to 26 letters. *Buddhivilāsinī*[2].

[2] *Līlāvatī* p.107

उदाहरणं शिल्पे–

*udāharaṇaṃ śilpe-*

Example on Architecture.

**Śloka** 116

एकद्वित्र्यादिमूषावहनमितिमहो ब्रूहि मे भूमिभर्तु–
र्हर्म्ये रम्येऽष्टमूषे चतुरविरचिते श्लक्ष्णशालाविशाले ।
एकद्वित्र्यादियुक्ता मधुरकटुकषायाम्लकक्षारतिक्तै–
रेकस्मिन्षड्रसैः स्युर्गणक कति वद व्यञ्जने व्यक्तिभेदाः ।। ११६ ।।

*ekadvitryādimūṣāvahana mitimaho brūhi me bhūmibhartu-*
*rharmye ramye´ṣṭamūṣe caturaviracite ślakṣṇaśālāviśāle .*
*ekadvitryādiyuktā madhurakaṭukaṣāyāmlakakṣāratiktai-rekasmin*
*ṣaḍrasaiḥ syurgaṇaka kati vada vyañjane vyaktibhedāḥ 116*

| | | | |
|---|---|---|---|
| अहो गणक ब्रूहि मे | एकद्वित्र्यादिमूषा वहनमिति | अष्टमूषे | श्लक्ष्णशाला विशाले |
| *aho gaṇaka brūhi me* | *ekadvitryādimūṣā vahanamiti* | *aṣṭamūṣe* | *ślakṣṇaśālā viśāle* |
| O mathematician tell me | the combinations of 1, 2, etc., windows, | from the 8 (windows) | in a spacious and pleasant quadrangle |

| | | |
|---|---|---|
| भूमिभर्तुः | हर्म्ये रम्ये | चतुरविरचिते |
| *bhūmibhartuḥ* | *harmye ramye* | *caturaviracite* |
| of a land lord's | beautiful palace | constructed by a skilled one. |

| | | |
|---|---|---|
| वद कति व्यक्तिभेदाः | स्युः एकस्मिन् व्यञ्जने | एकद्वित्र्यादियुक्ता |
| *vada kati vyaktibhedāḥ* | *syuḥ ekasmin vyañjane* | *ekadvitryādiyuktā* |
| Tell me (mathematician) | are there in a sauce | containing |
| how many combinations | | 1, 2, ... 6 at a time |

| | |
|---|---|
| षड्रसैः मधुरकटु– | कषायाम्लकक्षारतिक्तैः |
| *ṣaḍrasaiḥ madhurakaṭu-* | *kaṣāyāmlakakṣāratiktaiḥ* |
| out of the six flavours: | astringent, sour, |
| sweet, pungent | saltish and bitter. |

Bhāskarācārya gives the solution.

| | |
|---|---|
| मूषान्यासः | ८ ७ ६ ५ ४ ३ २ १ |
| *mūṣānyāsaḥ* | १ २ ३ ४ ५ ६ ७ ८ |
| Data for the window | *8 7 6 5 4 3 2 1* |
| arranged in 2 orders: | *1 2 3 4 5 6 7 8* |

| | |
|---|---|
| लब्धा | एकद्वित्र्यादिमूषावहनसंख्याः |
| ८ २८ ५६ ७० ५६ २८ ८ १ | १ २ ३ ४ ५ ६ ७ ८ |
| *labdhā* | *ekadvitryādimūṣāvahanasaṃkhyāḥ* |
| *8 8 56 70 56 28 8 1* | *1 2 3 4 5 6 7 8* |
| Obtained (from the above śloka) | |
| 8 28 56 70 56 28 8 1 | 1 2 3 4 5 6 7 8 |

| | | |
|---|---|---|
| एवमष्टमूषे | राजगृहे | मूषावहनभेदाः २५५ । |
| *evamaṣṭamūṣe* | *rājagṛhe* | *mūṣāvahanabhedāḥ 255 .* |
| Thus in the 8 windows | of king's palace | the total of all combinations is |
| | | $2^8$-1* = 255. |

* -1 corresponds to the combination with no window being open.

A similar solution is given by Bhāskarācārya for the second example.

# 10
# Progressions and Series

In this chapter we consider various progressions and the sums of the corresponding series such as partial sums of integers, sum of these sums, sums of squares and cubes, Arithmetic and Geometric progressions and their corresponding sums.

## 10.1 Partial Sum of Integers from one onwards and their Partial Sums

अथ श्रेढीव्यवहारः ।

*atha śreḍhīvyavahāraḥ*

Now about a series (of numbers).

*Buddhivilāsinī*[1] says

भिन्नं भिन्नं यत्किञ्चिद्द्रव्यादिकमेकीक्रियते तच्छ्रेढीत्युच्यते वृद्धैः । व्यावहारिकीयं संज्ञा ।

*bhinnaṃ bhinnaṃ yatkiñciddravyādikamekīkriyate tacchreḍhītyucyate vṛddhaiḥ vyāvahārikīyaṃ saṃjñā.*

Whatever quantities are added part by part, is said to be a series by the (learned) old. This is a definition for all practical purposes.

तत्र सङ्कलितैक्ये करणसूत्रं वृत्तम्–

*tatra saṅkalitaikye karaṇasūtraṃ vṛttam-*

Here is a śloka to find sum of the series.

**Śloka** 117

सैकपदघ्नपदार्धमथैकाद्यङ्कयुतिः किल सङ्कलिताख्या

सा द्वियुतेन पदेन विनिघ्नी स्यात्त्रिहृता खलु सङ्कलितैक्यम् ।। ११७ ।।

[1] *Līlāvatī* p.112