Brevity in Pāṇini's Aṣṭādhyāyī

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Introduction

How does a language communication take place between a speaker and a listener has intrigued Indian scholars and it resulted into the development of grammars and theories of verbal cognition. The important milestone in this development is the Pāṇini's grammar in the form of AṢṭādhyāyī (a book with eight chapters), dated around 500 BC. This book is in the form of compact rules and does not contain any prologue or foreword explaining what it is about. It is only through the Patañjali's MahābhāṢya (Great commentary) in around 200 BC, we come to know the purpose of this work. Pāṇini's grammar is known for its wide coverage of the grammar of the then prevalent Sanskrit. It is the only grammar for any language in the world with such a wide coverage. The debates on various aspects of this grammar continue even today and these debates thus have kept the tradition alive and the grammar extant.

In India, the grammar was born out of the necessity to protect the Vedas, the sacred texts. Several grammarians existed before Pāṇini and thus considerable amount of grammatical knowledge existed before him. He himself mentions a few grammarians in his AṢṭādhyāyī. The major contribution of Pāṇini was to compile all this earlier work, systematize it and further formalize it. Pāṇini's contribution to linguistics has some parallels with his contemporary Greek mathematician Euclid's contribution, who also systematized all the earlier knowledge of mathematics and formalized it in the form of the 'Elements'. The way 'Elements' influenced the development of Mathematics in Europe, in India Pāṇini's AṢṭādhyāyī influenced the later developments in the field of linguistics and other branches of knowledge especially the schools of philosophy.

The importance of *AṢādhyayī* is three fold. The first one as an almost exhaustive grammar for any human language with meticulous details

yet small enough to memorize. It is often admired for its simplicity and the completeness of its coverage of the then prevalent Sanskrit language. Presented in less than 4000 sutras (aphorisms) with around 7000 words, Bloomfield describes it as the 'greatest monuments of human intelligence'.

Many scholars of *AṢṭādhyāyī* believe that though *AṢṭādhyāyī* is written to describe the then prevalent Sanskrit language, it provides a grammatical framework which is general enough to analyse other languages as well. This is evident from the fact that this grammar has theoretically influenced the western linguistic theory (which is less than 2 century old) in many ways. The linguists admit that many of the insights of Pāṇini's grammar still remain to be captured. This makes the study of *AṢṭādhyāyī* important from the point of view of concepts it uses for language analysis.

The third aspect of *AṢṭādhyāyī* is its organization. The set of less than 4000 *sūtra*s is similar to any computer program, with one major difference, the program being written for a human being and not for a machine, thereby allowing some non-formal or semi-formal elements which require a human interpretation. This makes Pāṇini the foremost informaticien, 25 centuries before computers came into existence. Pāṇini paid utmost attention to the way information is coded in Sanskrit and used this insight not only to describe the grammar of Sanskrit but he also used these features in his Meta language to formulate the grammar of Sanskrit. The intricate system conventions governing rule interaction and rule application, the linear arrangement of partially ordered sets in the form of *śivasutras*, the linearized representation of hierarchical relationships, use of markers to trigger the application of sūtras are some of the techniques found in the organisation of Pāṇini's grammar. No wonder, the modern age of information theory has provided a new dimension to the studies of *AṢṭādhyāyī* from the perspective of information coding. Similar to the definitions, axioms, postulates and theorems in Euclid's Elements, in AṢṭādhyāyī we find a scientific treatment of language with sūtras providing definitions and postulating certain linguistic terms. We find here a consistent use of meta language. Majority of the sūtras deal with a step by step well defined procedure to derive word forms from the postulated root and a suffix, and new roots from the old ones. These procedures are all modular calling one or more sub-procedures to perform specific tasks. The whole system is very much similar to a computer programme

performing a complex task. Further, similar to expert systems such as a medical diagnostic system AṢṭādhyāyī has special rules to resolve the conflicts among the applicable rules. It also introduces a concept of *lopa* 'elision' which is very close to the concept of zero later found in the Indian mathematical texts. Pāṇini's grammar is known for its brevity. It is said of grammarians that[1] they rejoice over the saving of half a vowel as over the birth of a son. So much is brevity important to them. In natural languages, we often experience a tension between precision and brevity. In order to keep the size of the grammar small, Pāṇini invented several techniques without compromising on the precision. Use of sūtra style, factoring out common words, and use of pratyāhāras are some of them.

Much has been written about the first two aspects of the grammar. In this article I would give a glimpse of the third aspect with a focus on various techniques Pāṇini used or invented for bringing in brevity without compromising the precision.

Sūtra Style

The period in the Indian history when Pāṇini compiled AṢṭādhyāyī is characterised by a sūtra style composition. Several texts such as Brahmasūtra, Pigala's Chandaśāstra, Gautama's Nyāyasūtra, Patañjali's Yogasūtra, Kaṇāda's VaiśeṢikasūtra and Jaimini's Mīmāsāsūtra were composed in sūtra style. Sūtras are like mathematical formulae which carry bundle of information in a few words. For example, in mathematics, the Pythagorean theorem is expressed as, $a^2 + b^2 = c^2$, where *c* is the hypotenuse and *a* and *b* are the other two sides of a right angled triangle. Here the notation such as 2 written as a superscript stands for square, the '+' stands for the addition operation and '=' stands for 'equal to' help in expressing the verbose sentence compactly without any ambiguity. Just as anybody who knows English can not understand this mathematical equation without a special training in mathematics, similarly, to understand the Pāṇinian sūtras one needs some special training.

A sūtra[2] has the following six properties.

*1*ardhamātrālāghavena putrotsavaṃ manyante vaiyākaraṇaḥ – paribhāṢenduśekhara, ed. F. Keilhorn (Bombay, Indu Prakash Press, 1868), 115)

*2*alpākṢaram asandigdham sāravat viśvato mukham | astobham anavadyam ca sūtram sūtravidoviduḥ ||

1. alpākṢaram: has minimum number of words,
2. asandigdham: is unambiguous,
3. sāravat: contains essence of the topic about which it is meant,
4. viśvato mukham: is general or has universal validity,
5. astobham: does not have any meaningless words, and
6. anavadyam: is devoid of any faults.

The sūtras in Pāṇini's grammar are very small in size. 4,000 sūtras have only around 7,000 words with approximately 75,000 phonemes. That means on an average each sūtra is just 18-19 phonemes (sound units) long! The complete AṢṭdhyāyī fits in a book of about 40 pages. One such sample sūtra from AṢṭādhyāyī is *ikoyaṇaci (A 6.1.77*[3]*)*. Being written in a natural language, they are very easy to memorize. In those days the memorization was very crucial since the transmission of knowledge was through oral communication. These sūtras are of various types. Some of them provide definitions. For example, the sūtra *arthavadadhāturapratyayaḥ prātipadikam* (A1.2.45), provides the definition of a nominal root. The definition says, a meaningful string of letters which is not a verbal root and which is not a suffix is a nominal root. Then there are some sūtras which are called the adhikāra sūtras that stand for the title of a section. For example a sūtra *kārake (A1.4.23)* starts a section on kāraka, a 'case' denoting a relation among the words. Some sūtras are the meta rules that help one interpret the sūtras. For example, a sūtra 'tasmin iti nirdiṢṭe pūrvasya' (A 1.1.66) says, if there is a word in seventh case suffix in a sūtra, then the prescribed operation takes place to the left of that word. Another sūtra 'vipratiṢedhe param kāryam' (A 1.4.2) states that in case there is a conflict between sūtras, the latter rule prevails. Then there are sūtras which prescribe certain action when certain conditions are satisfied. For example, the sūtra 'karmaṇi dvitīyā' (A 2.3.2) says an object (karman) takes an accusative (second) case suffix, if it is not already expressed.

[3]The sūtras are numbered as A chapter.section.number, where 'A' stands for AṢṭādhyāyī, the chapter number is from 1 to 8 and section number is from 1 to 4. Thus A 1.3.2 stands for the second sūtra in the third section of the first chapter.

Anuvṛtti

In order to keep the size of his grammar small, Pāṇini used one more technique called 'anuvṛtti'. This technique is very close to the concept of factorization in Mathematics. During factorization, a term common to two product terms is factored out simplifying the expression. For example

a*b + a*c = a*(b+c).

Here the term 'a' common to the product terms a*b and a*c is factored out. This property is observed in languages as well. For example,

John went home.
He slept.

is succinctly expressed as

John went home and slept.

or the following two sentences

John went home.
Mary went home.

are expressed compactly as

John went home and Mary too.

Pāṇini took advantage of this phenomenon of the natural language in his grammar and thereby reduced the size of the grammar considerably. As an example, consider the following two sūtras.

upadeśe ac anunāsika it (A 1.3.2)
halantyam (A 1.3.3)

The first sūtra says, 'in the strings that are taught by Pāṇini (*upadeśe*), the nasalized (*anunāsika*) vowel (*ac*) is termed as 'it'. The next sūtra just says 'the consonant (*hal*) in the end (*antyam*)'. This is to be interpreted as 'in the strings that are taught by Pāṇini, the consonant at the end of a string is (also) termed as "it"'. Thus the words 'upadeśe' and 'it' are to be borrowed from the previous sūtras to interpret the latter sūtra. This process of borrowing of terms from the previous sūtras is called 'anuvṛtti'. This phenomenon is used very frequently by Pāṇini. Sometimes this borrowing continues for hundreds of sūtras. A sūtra may borrow from more than one previous sūtras. Such borrowings can be visually represented by proper indentation. The borrowing from multiple sūtras is typically nested properly. We give an example illustrating the anuvṛtti represented as nesting. Following 7 sūtras define the 'it' marker.

upadeśe ac anunāsika it (A 1.3.2)
halantyam (A 1.3.3)
na vibhaktau tusmā (A 1.3.4)
ādi ñiṭuḍavaḥ (A 1.3.5)
Ṣaḥ pratyayasya (A 1.3.6)
cuṭū (A 1.3.7)
laśaku ataddhite (A 1.3.8)

In these sūtras, the words 'upadeśe' and 'ac', though mentioned only in the first sūtra, need to be borrowed in all the following sūtras for the sake of completeness. So in grammarian's language, the words 'upadeśe' and 'ac' are said to have anuvṛtti till the sūtra A1.3.8. Similarly, the anuvṛtti of the word 'ādi' from A1.3.5 continues till A1.3.8. And the anuvṛtti of the word 'pratyayasya' from A1.3.6 continues till A1.3.8. If we show the anuvṛtti by indentation, these sūtras, then, look like

upadeśe it
  ac anunāsika
  halantyam; na vibhaktau tusmā
  ādi
    ñiṭuḍavaḥ
    pratyayasya
      Ṣaḥ
      cuṭū
      laśaku ataddhite

If we expand each sūtra by distributing the repeated words, we get the full form of the sūtras as shown below.

upadeśe ac anunāsika it
upadeśe halantyam; na vibhaktau tusmā **it**
upadeśe ādiḥ ñiṭuḍavaḥ **it**
upadeśe pratyayasya **ādiḥ** Ṣaḥ **it**
upadeśe pratyayasya ādiḥ cuṭu **it**
upadeśe ataddhita **pratyayasya ādiḥ** laśaku **it**

The anuvṛtti of the words from the previous sūtras are shown in **bold**. Total number of words after repeating them in each sūtra is 30 as

against 15 in the AṢṭādhyāyī. By using the technique of anuvṛtti, Pāṇini could achieve compression by a factor of 3 in terms of byte size (Kornai, 2008). This compression is very much important in the context of oral tradition. Hermann Ebbinghaus, who studied the behaviour of mental processes reported that the time required to memorize increases sharply as the text size increases. In other words the memorisation curve is very steep. The compression factor of 3 then becomes significant, given the fact that even to memorize around 4,000 sūtras a teenage student on average needs 6 months. The size of the AṢṭādhyāyī is thus just fit (optimal) for memorization.

Pratyāhāra sūtras

The book opens with a list of sounds in Sanskrit arranged in a special order (see Fig 1) different from their order in the normal Sanskrit alphabet (see Fig 2). This special sequence is known as Pratyāhāra sūtras or Māheśvarasūtras or Śivasūtras.

a i u **Ṇ**
ṛ ḷ **K**
e o **Ṅ**
ai ao **C**
h y v r **Ṭ**
l **Ṇ**
ñ m ṅ n **M**
jh bh **Ñ**
gh ḍh dh **Ṣ**
j b g ḍ d **Ś**
kh ph ch ṭh th c ṭ t **V**
k p **Y**
ś Ṣ s **R**
h **L**

Figure 1: Pratyāhāra sūtras

a ā i ī u ū ṛ ḷ e ai o au
k kh g gh ṅ
c ch j jh ñ
ṭ ṭh ḍ ḍh ṇ
t th d dh n
p ph b bh m
y r l v
ś Ṣ s h

Figure 2: Normal arrangement of sounds

Let us see now why the sounds were re-arranged by Pāṇini. Pāṇini needed 42 subsets of these phonemes in order to describe several phonetic changes that occur in a continuous speech, as well as during the word formation process. Some such sets required by him are

voiceless stops = {k,kh,c,ch,ṭ,ṭh,t,th,p,ph}

voiceless stops and spirants except h = {k,kh,c,ch,ṭ,ṭh,t,th,p,ph,ś,Ṣ,s }

voiceless unaspirated stops and spirants except h = {k,c,ṭ,t,p,ś,Ṣ,s }

voiceless unaspirated stops except velars and bilabials = {c,ṭ,t}

Now in order to define any operations on these sets, one needs to name them, and also one needs to remember which phoneme belongs to which sets. Defining and naming 42 such sets so that the association of members with the sets can be remembered is not an easy task.
In order to appreciate the concept, let me give an analogy. Imagine that you are an organising secretary of an event which has 42 sub-committees. The total number of volunteers available with you are also 42. Now each sub-committee has at least 2 members in it. This means a volunteer may belong to more than one sub-committee. As an organiser you would have to remember which volunteer belongs to which sub-committees and who are the members in each sub-committee. So it is natural for you to prepare a table with columns representing the committees and the rows representing the volunteers and put

appropriate check marks - cross or tick in every cell. Imagine a situation where you do not have access to a mobile or a computer or even any writing material where you could store this information and refer to it when needed. All that is available with you is your memory. Now, in order to remember which committees a volunteer belongs to, or to know all the members who belong to a given committee, one needs to remember this 42*42 table, 1764 bytes of information. Is there any better way of representation? Pāṇini did something radically different.

Let me take a small example with 6 members instead of 42, and explain what Pāṇini did. In a certain class in a school there are 6 students who participate in various games. Let us name them from 'a' to 'f'. The games they participate are as under.

Cricket: b, c

Tennis: d, e, f

Football: a, d, f

Chess: b, e, f

As is the case with all students, these students also like to gossip and chat whenever they get an opportunity. Hence students playing the same games want to sit next to each other. Now if these students sit in an alphabetic order then student 'b' can not talk with 'e' on chess, since he would be intervened by two other students 'c' and 'd'. Similarly 'a' will not be in a position to talk to his other friends 'd' and 'f' on football. However, if they are seated in the following order

c b e f d a

then everybody is happy. All the players having common interests are adjacent to each other. Now we introduce markers M_1, M_2, M_3 and M_4 indicating the boundary for Cricket, Chess, Tennis and Football players respectively.

c b M_1 e f M_2 d M_3 a M_4

The slice starting with 'c' and ending with M_1 gives us the names of players playing cricket. Let us name this slice by 'cM_1'. The slice '$b M_2$' gives the names of Chess players, the slice 'eM_3' gives the names of Tennis players, and finally 'fM_4' gives the names of football players. Such a naming scheme, which is a sort of abbreviation is called a **pratyāhāra**. In order to know the members of a set or to know the name of a set given the elements, one needs to memorise this linear list of 10 elements. That's all!

This is how Pāṇini solved his problem. He arranged 42 Sanskrit sounds in a linear order and inserted some markers (called *anubandha*s), in between that serve as the right boundary. Pāṇini needed 14 markers in order to describe all 42 subsets (See Figure 1)[4]. Following the naming scheme described above, 'aC' will correspond to the vowels 'a i u ṛ ḷ e o ai au'. Thus in order to know which sounds belong to a particular set, or given a set of sounds what is the name of that set, one needs to memorize just a linear list of 42+14 = 56 phonemes as against a 2 dimensional table of 42*42=1764 bytes. Reduction by a factor of 33!

One may ask a question that given a few subsets of a universal set, is it always possible to arrive at a linear order of the elements of the universal set so that one can retrieve the subsets as slices of this linear order? It has been proved mathematically (Petersen, 2009) that it is not necessary that one can always get such a linear order. Sometimes one may have to repeat one or more elements to get the linear order. For example, Pāṇini has repeated a phoneme 'h' in order to get such a linear order. And this is the reason there are an 57 phonemes in the above arrangement in Fig. 1, and not 56!

This arrangement of phonemes in śivasūtras

1) deviates from the normal order of sounds in Sanskrit,

2) has 'h' sound repeated in the fifth and the fourteenth row, and

3) has the marker **Ṇ** repeated in the first and the sixth row.

These features have intrigued the researchers since long. There have been several attempts by the scholars to account for this arrangement on the basis of historic grounds, linguistic grounds and even mathematical grounds. While historic and linguistic grounds justify the arrangement and the repetition of 'h' sound, the mathematical account even proves the optimality of this arrangement with repetition of 'h'. While the repetition of 'h' was necessary in order to describe all the needed sets, the repetition of the marker 'Ṇ' makes the name of a set ambiguous. For example, the set named 'aṆ' may correspond to either '{a, i, u}' or '{a,i,u,ṛ,ḷ,e,o,ai,ao,h,y,v,r,l}'. Similarly the set named 'iṆ' is ambiguous between the two sets '{i,u}' and '{i,u,ṛ,ḷ,e,o,ai,ao,h,y,v,r,l}'. Patañjali in his MahābhāṢya (Great Commentary) asks a question, was there any dearth of symbols that

*4*There are legands associated with the origin of this arrangement. It is believed that Lord Śiva played his ḍamaru (a small instrument with strings attached to it, played by holding it in a palm) and the pratyāhāra sūtras emerged from his ḍamaru. The arrangement resembles the shape of a ḍamaru. Probably this might be the reason behind calling them Śivasūtras or Maheśvarasūtras!

Pāṇini repeated the marker? He discusses on the repetition of the marker Ṇ, and illustrates how the ambiguity can be resolved in all cases. Thus the fact that Pāṇini repeated the marker **Ṇ**, forces a reader to think before interpreting, and this thinking leads to a specific conclusive episode of knowledge. Therefore, Patañjali concludes that this repetition of a marker should not be considered to be a reason for any doubt[5].

Meta Language

Pāṇini has used meta rules and meta language in the design of his AṢṭādhyāyī. An important aspect of this meta language is special meanings he associates with the case markers. Let me explain this with an example. Pāṇini needed rules of the form *B* changes to *C* if it is preceded by *A* and followed by *D*. This may be succinctly written as

$$A\ [\ B \rightarrow C\ ]\ D$$

Now, in order to convey this rule orally, one needs to fix the positions of *A*, *B*, *C* and *D*. However, by nature, Sanskrit is a free word language. Pāṇini did not compromise with the 'free' ness language enjoys while formulating his sūtras. Instead, he redefined the meanings of case suffixes to code the information of relative position. Pāṇini defines technical meanings of case suffixes with the following meta rules.

- A word in genitive case represents an item that undergoes a change[6].
- A word in locative case suffix[7] indicates that the change will occur in a word that is to its left, and
- the word with ablative case suffix[8] indicates that the change will take place in a word that is to the right of it.

Now we explain a sūtra from Pāṇini where he uses the pratyāhāras as well as the meta rules to state a rule of sandhi. The sūtra is

iko yaṇaci (A 6.1.77).

This sūtra has three words in it 'ikaḥ', 'yaṇ', and 'aci'. These words are the genitive form of 'ik', nominative form of 'yaṇ' and the locative form of 'ac' respectively. So effectively the analysed form of the sūtra is

5 vyākhyānataḥ viśeṢa pratipattiḥ na hi sandehāt alakṢaṇam.
6 ṢaṢh sthāneyogā (A 1.1.49).
7 tasmin iti nirdiṢe pūrvasya (A 1.1.66).
8 tasmāt iti uttarasya (A 1.1.67).

ik{6} yaṇ{1} ac{7}

Now, the terms ‘ik’ ‘yaṇ’ and ‘ac’ are the pratyāhāras which stand for the following set of phonemes.

ik = { i u ṛ ḷ }
yaṇ = { y v r l }
ac = all vowels
= { a i u ṛ ḷ e o ai ao }

Following the meta language, now this rule states that

[{ i u ṛ ḷ }→ { y v r l }] { a i u ṛ ḷ e o ai au }

That is, the vowels ‘i’, ‘u’, ‘ṛ’ and ‘ḷ’ change to the semi vowels ‘y’, ‘v’, ‘r’, ‘l’ respectively when followed by a vowel. For example, iti + atra → ityatra. Which word in the sūtra tells us that the replacement is to be carried out ‘respectively’? Well, there is no word in the sūtra which states this. But there is another sūtra *sthāne’ntaratamaḥ (A1.1.50)* which states that when the replacement operation is specified, and the number of elements in the set denoting the ‘replaceable’ elements and the number of elements in the set that will replace it are same, then the replacement is to be interpreted to be ‘respectively’.

This was an example to illustrate how Pāṇini achieved brevity in stating the sūtras with the help of pratyāhāra and the meta language. Thus, the rule “the vowels ‘i’, ‘u’, ‘ṛ’ and ‘ḷ’ change to the semi vowels ‘y’, ‘v’, ‘r’, ‘l’ respectively when followed a vowel” is expressed succinctly by Pāṇini as ‘ikoyaṇaci’.

Here is a word of caution. The use of meta language brings in more challenges in the interpretation. Because, Pāṇini mixes the normal Sanskrit language with the meta language in sūtras. Thus an interpreter has to ‘think’ before deciphering the meaning of the sūtras. One can not mechanically interpret them.

Conclusion

With the emergence of Linguistics, linguists had started recognising the importance of Pāṇini’s grammar. And now with the advent of computer technology, computational linguists have started recognising Pāṇini as an information scientist. A group of young researchers is looking at the way Pāṇini has framed the rules, the meta language he used, conflict resolution techniques he used for application of rules, and so on. Another group is looking at the ‘universal’ features in his grammar that can be used for modeling other languages. Thus while the debates on Pāṇini’s grammar have kept it extant, researchers throwing light on

various new aspects of Pāṇini's grammar just remind us of the story of an elephant being described by the blind men!

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